Optimal Fiscal Policy With Heterogeneous Agents and Capital: Should We Increase or Decrease Public Debt and Capital Taxes?*

François Le Grand Xavier Ragot†

September 20, 2024

Abstract

We analyze optimal fiscal policy in a heterogeneous-agent model with capital accumulation and aggregate shocks, where the government uses public debt, a capital tax, and a progressive labor tax to finance public spending. We first study a tractable model and show that the steady-state optimal capital tax can be positive if credit constraints are occasionally binding. However, the existence of such an equilibrium depends on the shape of the utility function. We also characterize the optimal dynamic of public debt after a public spending shock. We confirm these findings by solving for optimal policy in a general heterogeneous-agent model.

Keywords: Heterogeneous agents, optimal fiscal policy, public debt

JEL codes: E21, H21, E44, D31.

---

*A very early version of this paper was circulated under the title "Optimal policy with heterogeneous agents and aggregate shocks." We are grateful to Andrew Atkeson (the editor), and three anonymous referees for their detailed comments. We also thank Stefania Albanesi, Manuel Amador, Adrien Auclert, Marco Bassetto, Anmol Bhandari, Javier Bianchi, Edouard Challe, Wouter den Haan, Jonathan Heathcote, Greg Kaplan, Christos Koulovatianos, Dirk Krueger, Felix Kubler, Albert Marcet, Juan Pablo Nicolini, Elisabeth Prohl, Morten Ravn, Vincent Sterk, Stephanie Schmitt-Grohe, Martin Uribe, Gianluca Violante, Pierre-Olivier Weill, three anonymous referees, and participants in seminars at the European University Institute, Minneapolis Federal Reserve, National Bank of Belgium, New York University, University of Kyoto, University of Luxembourg, University of Zurich, as well as the EEA, NBER SI (MPF), SED, and T2M conferences. We also acknowledge financial support from ANR (ANR-20-CE26-0018 IRMAC). This paper was edited by Andrew Atkeson.

†Le Grand: Rennes School of Business; `francois.le-grand@rennes-sb.com`. Ragot: SciencesPo-CNRS, OFCE, and CEPR; `xavier.ragot@sciencespo.fr`.

1 Introduction

What are the optimal levels of public debt and capital taxes? After a positive public spending shock, should the government temporarily increase capital taxes, or the progressivity of the tax system? These long-standing questions are likely to remain relevant in many countries in the coming years, as policymakers increasingly discuss additional public spending to combat climate change or for military purposes. Such questions require considering both the distortionary and redistributive effects of tax changes, while also taking into account general equilibrium effects. Heterogeneous-agent models in the tradition of the Bewley–Huggett–Imrohoroğlu–Aiyagari literature (Bewley, 1983; Imrohoroğlu, 1989; Huggett, 1993; Aiyagari, 1994; Krusell and Smith, 1998) are relevant tools for analyzing these questions, because they generate a realistic amount of heterogeneity along with general and dynamic equilibrium effects. However, after seminal papers investigating optimal fiscal policy in these environments (Aiyagari, 1995; Aiyagari and McGrattan, 1998), the literature has mainly moved towards a positive analysis. Little is known about the optimal levels and dynamics of public debt and capital taxes, because of both the theoretical and the computational difficulties of solving for optimal fiscal policy.

This paper analyzes optimal fiscal policy in heterogeneous-agent models, considering capital accumulation, progressive labor income taxation, capital taxation, public debt, and aggregate shocks. The only frictions considered are incomplete markets for idiosyncratic risk, occasionally binding credit constraints (which appear to be the key friction), and the given set of fiscal instruments. In particular, the planner in this model cannot use lump-sum taxes, which are known to potentially restore Ricardian equivalence in some settings (Bhandari et al., 2017). Ultimately, we find and characterize equilibria that feature optimal positive levels of both capital taxation and public debt. While our analysis admittedly abstracts from other frictions, such as nominal rigidities or frictional labor markets, we identify new mechanisms that will also be present in more general environments with these features.[1] Our paper makes three specific contributions, which we now discuss in more detail.

To understand the optimal steady-state levels of capital taxation and public debt, we first consider a simple heterogeneous-agent model and a utilitarian planner with

---

[1] Considering only price or wage stickiness would yield the same allocation as in our economy, since the only role of optimal monetary policy is price stability, given the set of fiscal instruments we consider (see LeGrand et al., 2024). Moreover, considering capital accumulation allows one to characterize the optimal dynamics of the capital tax and to discuss its relationship with the results of the vast Chamley–Judd literature on optimal capital taxation.

full commitment. In this simple model, two types of agents face deterministic income fluctuations between employment and unemployment, as in Woodford (1990). The only financial frictions are credit constraints, which can occasionally be binding, although there is no uninsurable idiosyncratic risk. Our first contribution is to prove that, contrary to the results of Judd (1985) and Chamley (1986), the steady-state optimal capital tax can be positive. However, it depends crucially on the shape of the utility function and on the intertemporal elasticity of substitution (hereafter IES). The intuition for this result is that savings induce a price externality, which the planner internalizes via a positive capital tax. The planner adjusts public debt so that the marginal return of capital equals the discount rate at the steady state. This is the well-known modified golden rule of Aiyagari (1995). With respect to savings, an extra unit saved increases the capital tax base and thus relaxes constraints on public finances in proportion to the capital tax. Absent any costs, the planner would like to set this marginal benefit to zero and maximize benefits – and hence to set a zero capital tax. Because the planner chooses savings that are consistent with the agents' Euler equations, the envelope theorem implies that there is no direct welfare effect for agents, and the costs on agents occur solely through a price externality of savings on post-tax interest rates and wages. When credit constraints are binding, larger aggregate savings require higher post-tax factor prices (because of the Euler equation and the first-order condition on labor), and thus lower tax rate, which reduces the tax return. In brief, when setting aggregate savings, the planner trades-off the benefits in terms of larger fiscal base with the costs in terms of higher post-tax factor prices.

When the utility function is separable, the price externality is related to the elasticity of interest and wage rates to savings, which are pinned down by the Euler equation and the labor supply first-order condition. The savings externality is then found to be proportional to the gap between the inverse of the IES of employed and unemployed agents. This makes the shape of the utility function key to setting the optimal capital tax. In particular, the optimal capital tax is always zero in the standard case of a separable, Constant Relative Risk Aversion (CRRA) utility function, for which the gap between IESs is zero. This is consistent with the results of Chamley-Judd. In that case, the net effect of aggregate savings on post-tax prices is null and there is no savings externality. The planner can simply choose savings that maximize its resources, and hence maintain a capital tax of zero. This result is also consistent with the claim of Chien and Wen (2024) and the numerical investigation of Auclert et al. (2022).

However, the steady-state capital tax can be positive if the utility function deviates from a constant IES. This is the case, for example, with separable DRRA (e.g., Stone-Geary or

Fishburn) utility functions. For non-separable utility functions, the elasticity of the factor prices with respect to savings includes an additional term related to the cross-derivative of the utility function with respect to consumption and labor supply. This new term explains why the optimal capital tax is positive for the Greenwood-Hurcowitz-Huffman (GHH) or the King-Plosser-Rebelo (KPR) utility functions.

Our second contribution is to fully characterize the conditions for the existence of an equilibrium in this environment with both positive optimal capital taxation and positive optimal public debt. To study existence, we consider a GHH-type utility function, such as in Aiyagari (1995), Diamond (1998) or Açikgöz et al. (2022). The existence of the steady-state equilibrium relies on three independent conditions: a non-first-best condition, a so-called Straub–Werning condition, and a standard Laffer condition. The Straub–Werning condition is based on Straub and Werning (2020) and states that public spending must be low enough to ensure a stationary steady state, and to avoid a situation in which the planner chooses to continuously reduce the capital stock (despite being able to raise enough resources in the steady state). Finally, a Blanchard–Kahn condition, ensures the stability of the equilibrium. Alongside a positive capital tax, the optimal fiscal system can be characterized (for some parameter combinations) as featuring a positive public debt, when saving is higher than the optimal capital stock.We extend this simple setting to an economy with ex-ante heterogeneity and show that the optimal capital tax depends on the social weights of the Ramsey planner. This result again differs from that of Judd (1985).

Equipped with these results, we analyze the optimal dynamics of fiscal policy after a one-time positive shock to public spending (a so-called MIT shock). Our third contribution is to show that, for a given net present value (NPV) of public spending, public debt increases (resp. decreases) when the persistence of the shock is low (resp. high). Consequently, the persistence of the shock is a key driver of the optimal dynamics of public debt. The intuition for these results is the following. In contrast to the complete-market case where agents initially hold some capital, in the incomplete-market model, the capital tax is not used to fully front-load the adjustment because taxing capital reduces the ability of agents to self-insure when markets are incomplete. In addition, in this type of model, public debt converges to its optimal steady-state value for any transitory shock to public spending. Consequently, if the shock's persistence is high, a transitory increase in public debt would require a welfare-reducing, highly persistent increase in taxes to finance public spending and reduce public debt. Therefore, the optimal policy is to front-load the adjustment and to temporarily reduce the public debt. When persistence is low, in contrast, the increase in public debt improves consumption smoothing, and a small increase in taxes is sufficient to ensure that public debt converges. The claim that optimal public debt can fall after a persistent public spending shock is already made in Feldstein (1985), who introduced a quadratic tax adjustment cost. Compared to this seminal literature, the current paper provides a micro-foundation for the cost of tax changes, based on distributional considerations, and generates an optimal long-run level of public debt.

Finally, we verify that the previous results, obtained in a stylized model, still hold in a realistic quantitative model. In this model, ex-ante different types of agents, all endowed with a GHH utility function, face heterogeneous uninsurable income risk. The planner aims to finance public spending through a capital tax, a nonlinear labor tax à la Heathcote et al. (2017), and public debt. The planner's Social Welfare Function (SWF) assigns weights to agents that depend on their ex-ante type. Our quantitative strategy is, first, to use an inverse-optimal approach to identify the SWF weights from the observed fiscal system, which is assumed to be an optimal steady state (as in Heathcote and Tsujiyama, 2021 among others). Second, we verify that the existence conditions identified in the analytical model also hold in the general model. Third, using the identified SWF, we compute the optimal dynamics of the capital tax, the labor tax, progressivity, and public debt after an (MIT) public spending shock. This strategy allows us to simulate the dynamics around a quantitatively relevant steady state. The results of the quantitative model are consistent with those of the theoretical model. Public debt increases when the persistence of the public spending shock is low, and decreases otherwise. The quantitative model also provides additional results. The optimal progressivity of the labor and the capital tax both increase after a positive public spending shock, but the increase is smaller when the persistence of the shock is higher. Optimal public debt also exhibits persistent deviations that are quantitatively much larger than that of other variables.

This paper is related to the literature on optimal fiscal policy in heterogeneous-agent models.[2] As mentioned above, the existence of well-defined steady-state Ramsey equilibria is still an open question. Conesa et al. (2009) considered transitions with constant instruments. Chien and Wen (2024) and Auclert et al. (2022) find that the Ramsey steady-state equilibrium does not exist for separable CRRA utility functions. Dyrda and Pedroni (2022) quantitatively solved for optimal policy by considering the full path of the policy instruments and using a KPR utility function. Aiyagari (1995) and Açikgöz et al. (2022) analyze optimal public debt when there is no wealth effect on labor supply. Bassetto and Cui (2024) study an environment where public debt can relax the producer's

---

[2] A large literature provides a positive analysis of fiscal policy in heterogeneous-agent models (e.g., Floden, 2001; Heathcote, 2005; Rohrs and Winter, 2017; Ferriere and Navarro, 2023, among many others).

credit constraint. They find that optimal steady-state capital taxes are positive, when public debt is constrained to be at the top of the Laffer curve. Our results show that these apparently contradictory results are consistent, as they are based different utility functions.[3]

Analyzing optimal fiscal policy in such an environment obviously relies heavily on results about idiosyncratic risk in complete-market economies.[4] As mentioned above, incomplete-market models allow for consideration of optimal positive steady-state capital taxation and redistribution. A recent literature reports the development of tools for solving for optimal policies with heterogeneous agents involving mostly monetary policy, for which the steady-state allocation is simpler to characterize, as optimal inflation is null (e.g., Bhandari et al., 2021; Acharya et al., 2023; LeGrand et al., 2024; Nuño and Thomas, 2022, among others). We use the truncation approach of LeGrand and Ragot (2022a), using the refinement of LeGrand and Ragot (2022b) to solve the curse of dimensionality. This method builds on the factorization method introduced by Marcet and Marimon (2019) and allows one to easily simulate models with many instruments and aggregate shocks. Because it is relatively new, we summarize the method in Section 4 below.

The rest of this paper is organized as follows. In Section 2, we present the general environment. In Section 3, we present simplifying assumptions and solve the tractable model. We present the general model and derive optimality conditions in Section 4. In Section 5 we calibrate and simulate the general model. We conclude in Section 6.

2 The Environment

Time is discrete and indexed by $t = 0, 1, \ldots$, and the economy is populated by a continuum of F heterogeneous types of agents. The type of an agent determines the dynamics of the productivity risk it faces. Each type f is distributed along a set I^f with measure ℓ^f. We follow Green (1994) and assume that the law of large numbers holds. There is a share m^f of type f, where $\sum_{f=1}^{F} m^f = 1$ and the population of each type is one, such that $\sum_{f=1}^{F} m^f \int_i \ell^f(di) = 1$.

Furthermore, the economy features production and a benevolent government that raises

---

[3] Albanesi and Armenter (2012) provide general sufficient conditions for the optimal steady-state capital tax to be zero in many environments. These conditions are not fulfilled in our setup for relevant cases, because the planner would need to use a distorting labor tax to finance public spending when the capital tax is zero, preventing the economy from converging to the first-best allocation.

[4] For relevant contributions, see Barro (1979); Chari et al. (1994); Farhi (2010); Bassetto (2014); Chari et al. (2020); Straub and Werning (2020); Collard et al. (2023) among others.

distorting taxes and public debt to finance an exogenous stream of public spending.

Risks. The aggregate shock solely affects public spending, denoted by $(G_t)_{t\geq 0}$, and is therefore assimilated to a public spending shock. We discuss below, in Section 5.4, the outcome of the model for other shocks. Furthermore, we assume that the whole path of public spending $(G_t)_{t\geq 0}$ becomes known to all agents in period 0. We will solve for the optimal adjustment of the economy after such a shock (often called an MIT shock), assuming that the planner cannot renege on their past commitments (See Section 4.4 below for further discussion).

Each agent's type f differs according to its productivity process. Each productivity process is a first-order Markov chain characterized by a finite set of productivity levels $\mathcal{Y}^f$ and a transition matrix Π^f. For the sake of simplicity, we assume that the number of possible productivity levels is the same for all types, and denoted by J – such that all transition matrices have the same dimension $J \times J$. We assume that each productivity process admits a unique stationary distribution that is denoted by the vector S_y^f, verifying $S_y^f = (S_y^f)^\top \Pi^f$.[5] In period t, the productivity of agent i of type f is $y_{i,t}^f$ and they will earn a before-tax labor wage $\tilde{w}_t y_{i,t}^f l_{i,t}^f$, where $l_{i,t}^f$ denotes their labor supply and $\tilde{w}_t$ the before-tax hourly wage. Their whole history of shocks up to t is denoted by $y_i^{f,t} := \{y_{i,0}^f, ..., y_{i,t}^f\}$.

Production. The production sector is standard. The consumption-investment goods of the economy are produced by a profit-maximizing representative firm. At any date t, the firm's production function combines labor L_t and capital K_{t-1}—which must be installed one period in advance—to produce Y_t units of the consumption goods. The production function is assumed to be of the Cobb-Douglas type, featuring constant returns to scale and capital depreciation. The total factor productivity is normalized to one. Formally, net-of-depreciation production is defined as

$$Y_t = F(K_{t-1}, L_t) = K_{t-1}^{\alpha} L_t^{1-\alpha} - \delta K_{t-1},$$

where $\alpha \in (0,1)$ is the capital share and $\delta \in (0,1)$ is the capital depreciation rate.

The firm rents labor and capital at respective factor prices $\tilde{w}_t$ and $\tilde{r}_t$. The profit maximization conditions of the firm imply the following expressions for factor prices:

$$\tilde{w}_t = F_{L,t} \text{ and } \tilde{r}_t = F_{K,t}, \tag{1}$$

---

[5] In the quantitative analysis of Section 4, the Markov chain can be shown to be irreducible and aperiodic; hence S_y^f is known to exist and to be unique.

where we use $F_{L,t} := F_L(K_{t-1}, L_t)$ and $F_{K,t} := F_K(K_{t-1}, L_t)$ to lighten notations.

Assets. In addition to capital, the economy also features public debt, whose size is denoted by B_t in period t. Public debt consists of one-period bonds issued by a benevolent government, which are assumed to be default-free. Because of our assumption of MIT shocks, there is no aggregate risk in this economy. Both capital and public debt are thus perfect substitutes, and no-arbitrage implies that they must pay the same after-tax return. Agents' savings, denoted by $(a^f_{i,t})_{i,f}$ at date t, are restricted to remain greater than an exogenous limit $-\underline{a} \leq 0$.

Period 0. We assume that the economy starts in period -1 with an endowment of wealth and productivity $(a^f_{i,-1}, y^f_{i,0})_i$ drawn from a distribution Λ_0, a given amount of public debt B_{-1}, and a given amount of capital K_{-1}, which together satisfy $K_{-1} + B_{-1} = \sum_{f=1}^F m^f \int_i a^f_{i,-1}\ell(di)$. The MIT shock is the public spending path $(G_t)_{t\geq 0}$, which is revealed at period -1 before households actually perform their portfolio choice. As a consequence, the two assets must also have the same after-tax return in period 0. The before-tax real interest rate between period -1 and period 0 is denoted $\tilde{r}_0$, and the MIT shock affects the allocation from period 0 onward.

Government. A benevolent government has to finance the exogenous stream of public spending $(G_t)_{t\geq 0}$ by levying distorting taxes on capital and labor, and by issuing public debt. The tax on capital is linear with a rate $(\tau^K_t)_{t\geq 0}$, and is actually levied on all interest bearing assets (capital and public debt). The tax on labor income is assumed to be nonlinear and possibly time-varying. We denote by $T_t(\tilde{w}yl)$ the amount of labor tax paid at date t by an agent earning the pre-tax labor income $\tilde{w}yl$. We follow Heathcote et al. (2017) (hereinafter HSV) and consider the following functional form:

$$T_t(\tilde{w}yl) := \tilde{w}yl - \kappa_t(\tilde{w}yl)^{1-\tau_t}, \tag{2}$$

where κ_t captures the level of labor taxation and τ_t the progressivity. Both parameters are assumed to be time-varying and will be the planner's instruments in the general model. When $\tau_t = 0$, labor tax is linear with rate $1 - \kappa_t$; oppositely, $\tau_t = 1$ corresponds to full income redistribution, where all agents earn the same post-tax income κ_t. Functional form (2) combined with the linear capital tax allows one to realistically reproduce the actual US system and its progressivity (see Heathcote et al., 2017 or Ferriere and Navarro, 2023).[6]

Combining the above elements, the government budget constraint can be written as:

$$G_t + (1+\tilde{r}_t)B_{t-1} \leq \sum_{f=1}^{F} m^f \int T_t(\tilde{w}_t y^{i,f} l_{i,t}^f)\ell^f(di) + \tau_t^K \tilde{r}_t (B_{t-1} + K_{t-1}) + B_t. \tag{3}$$

This states that public spending and past public debt repayment can be financed out of the proceedings of labor and capital taxation, as well as by the issuance of new public debt. To simplify the government budget constraint, in the spirit of Chamley (1986) we introduce generalized post-tax factor prices, which are denoted without a tilde. We define the gross and net interest rates r_t and R_t, respectively, and the wage rate w_t as:

$$w_t := \kappa_t (\tilde{w}_t)^{1-\tau_t}, \tag{4}$$

$$R_t := 1 + r_t = 1 + (1-\tau_t^K)\tilde{r}_t. \tag{5}$$

The model can be expressed analytically using the pair of post-tax rates (R_t, w_t) rather than pre-tax ones $(\tilde{r}_t, \tilde{w}_t)$, which simplifies the algebra. The values of the fiscal instruments τ_t^K, κ_t, and τ_t can then be recovered from the allocation. Using the constant return-to-scale property of the production function, the governmental budget constraint (3) becomes:

$$G_t + R_t B_{t-1} + w_t \sum_{f=1}^{F} m^f \int_i (y_{i,t}^f l_{i,t}^f)^{1-\tau_t} \ell^f(di) \leq F(K_{t-1}, L_t) - (R_t - 1)K_{t-1} + B_t, \tag{6}$$

which can be interpreted by observing that total output and new public debt are used to finance public spending, past public debt repayment, post-tax capital rents, and post-tax wages. In the constraint (6), the effect of factor supplies on aggregate output is fully internalized by the government. Equation (6) can indeed be interpreted by viewing the planner as having the economy's output and newly issued public debt as revenue, and paying back old debt, public spending, and factor supplies with post-tax rates. This has two implications, which matter for the discussion of the planner's choices in Section 3.1. First, it explains why zero capital tax, which maximizes output, also relaxes the budget constraint the most. Indeed, if everything else were constant, including R_t, the level of the capital stock that maximizes the right hand side of (6) would satisfy $1 + F_{K,t} = R_t$, which corresponds to zero capital tax – as can be seen from equations (1) and (5). Second,

---

[6] The literature uses either the combination of a linear tax and a lump-sum transfer (e.g., Açikgöz et al., 2022; Dyrda and Pedroni, 2022) or the HSV structure. Heathcote and Tsujiyama (2021) showed that the HSV structure is quantitatively more relevant. However, we show in Appendix A.9 that our results still hold in the presence of an affine tax system.

equation (6) also shows that, if everything else were constant, higher post-tax interest and wage rates have a negative impact on the governmental budget constraint. Indeed, the planner would have to transfer a larger amount of resources to agents, leaving fewer resources to finance public spending.

Agents' program and resource constraints. At each date t, agents consume goods in quantity c_t, supply labor in quantity l_t, and save an amount a_t. They derive an instantaneous utility from consumption and labor supply denoted by $U(c_t, l_t)$; the utility function will be specified later. The discount factor is constant and denoted by $\beta \in (0, 1)$.

Using the post-tax rate definition (4), the post-tax labor income of an agent i of type f amounts to $\tilde{w}_t y^f_{i,t} l^f_{i,t} - T_t(\tilde{w}_t y^f_{i,t} l^f_{i,t}) = w_t (y^f_{i,t} l^f_{i,t})^{1-\tau_t}$, while post-tax capital income is equal to $R_t a_{i,t-1}$. Formally, the program of agent i of type f endowed with the given initial wealth $a^f_{i,-1}$ can be expressed as:

$$\max_{\{c^f_{i,t}, l^f_{i,t}, a^f_{i,t}\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c^f_{i,t}, l^f_{i,t}), \tag{7}$$

$$c^f_{i,t} + a^f_{i,t} = R_t a^f_{i,t-1} + w_t (y^f_{i,t} l^f_{i,t})^{1-\tau_t}, \tag{8}$$

$$a^f_{i,t} \geq -\underline{a},\ c^f_{i,t} \geq 0,\ l^f_{i,t} \geq 0. \tag{9}$$

Note that because of our assumption of MIT shocks, the expectation operator in (7)—as well as in the rest—solely concerns idiosyncratic shocks. Equation (8) is the budget constraint, and inequalities (9) are the credit constraint and the non-negativity constraints.

The solution of the previous program is a set of policy rules defined over the product space of productivity histories and initial asset holdings: $c^f_t : (\mathcal{Y}^f)^t \times [-\bar{a}; +\infty) \to \mathbb{R}^+$, $a^f_t : (\mathcal{Y}^f)^t \times [-\bar{a}; +\infty) \to [-\bar{a}; +\infty)$, and $l^f_t : (\mathcal{Y}^f)^t \times [-\bar{a}; +\infty) \to \mathbb{R}^+$. To lighten the notation, we will simply write $c^f_{i,t}$, $a^f_{i,t}$, and $l^f_{i,t}$ (instead of $c^f_t(y^{f,t}_i, a^f_{i,-1})$, $a^f_t(y^{f,t}_i, a^f_{i,-1})$, and $l^f_t(y^{f,t}_i, a^f_{i,-1})$) and use the same notation for all variables.[7]

Denoting by $\beta^t \nu^f_{i,t} \geq 0$ the Lagrange multiplier on the agent's credit constraint, the consumption Euler equation can be written as

$$U_c(c^f_{i,t}, l^f_{i,t}) = \beta \mathbb{E}_t \left[R_t U_c(c^f_{i,t+1}, l^f_{i,t+1}) \right] + \nu^f_{i,t}, \tag{10}$$

where we denote by U_c and U_l the first-order derivatives with respect to c and l, and by

---

[7] Hence, the aggregation of the variable X_t in period t will be written as $\int_i X^f_{i,t} \ell^f(di)$ instead of the more involved explicit notation $\int_{a^f_{-1}} \sum_{y^{f,t} \in \mathcal{Y}^t} \theta^f_t(y^{f,t}) X(y^{f,t}, a^f_{-1}) d\Lambda_0(a^f_{-1}, y^f_0)$, where $\theta^f_t(y^{f,t})$ is the probability of the occurrence of history $y^{f,t}$ in period t for an agent of type f.

U_{xy} with $(x, y = c, l)$ the second-order derivatives.

The first-order condition (FOC) on labor is:

$$-U_l(c^f_{i,t}, l^f_{i,t}) = (1-\tau_t)w_t y^f_{i,t}(y^f_{i,t}l^f_{i,t})^{-\tau_t}U_c(c^f_{i,t}, l^f_{i,t}), \tag{11}$$

and the clearing of financial and labor markets implies the following equalities:

$$A_t = K_t + B_t \text{ and } \sum_{f=1}^{F} m^f \int y^f_{i,t}l^f_{i,t}\ell^f(di) = L_t. \tag{12}$$

The clearing of the goods market reflects the fact that the sum of aggregate consumption, public spending, and the new capital stock balances the production output and past capital:

$$\sum_{f=1}^{F} m^f \int_i c^f_{i,t}\ell^f(di) + G_t + K_t = K_{t-1} + F(K_{t-1}, L_t). \tag{13}$$

The Social Welfare Function. The planner considers a weighted sum of agents' intertemporal utilities, where the social weight of each agent (sometimes referred to as Negishi or Pareto weights) is denoted by ω^f and depends solely on their time-invariant type. The utilitarian case corresponds to $\omega^f = 1$ for all f. The aggregate social welfare W_0 can thus be written as:

$$W_0 = \sum_{f=1}^{F} m^f \omega^f \left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_{i\in I^f} U(c^f_{i,t}, l^f_{i,t})\ell^f(di)\right), \tag{14}$$

where the term between brackets is the sum, over the whole population I^f, of the ex-ante intertemporal utilities of agent i of type f. The planner attributes a social weight ω^f to each agent, corresponding to the agent's type. In addition, the sum of the weighted utilities of type f agents is itself weighted by the share m^f of type f in the total population. We normalize the social weights to sum to 1: $\sum_{f=1}^{F} m^f\omega^f = 1$.

Equilibrium definitions The Ramsey problem with full commitment consists in finding the fiscal policy that delivers the competitive equilibrium with the highest aggregate social welfare. We start with the formal definition of a competitive equilibrium.

Definition 1 (Competitive equilibrium (CE)). *A competitive equilibrium is a collection of individual variables* $(c^f_{i,t}, l^f_{i,t}, a^f_{i,t})^{i\in\mathcal{I}^f, f=1,\ldots,F}_{t\geq 0}$*, aggregate quantities* $(K_t, L_t, Y_t)_{t\geq 0}$*, prices* $(\tilde{w}_t, \tilde{r}_t)_{t\geq 0}$*, fiscal policy* $(\tau^K_t, \kappa_t, \tau_t, B_t)_{t\geq 0}$*, and public spending* $(G_t)_{t\geq 0}$ *such that for an initial distribution of wealth and productivity* $(a^f_{i,-1}, y^f_{i,0})_{i\in\mathcal{I}^f, f=1\ldots F}$ *and for initial values of* *capital stock and public debt verifying* $K_{-1} + B_{-1} = \sum_{f=1}^{F} m^f \int_i a_{i,-1}^f \ell^f(di)$*, the following holds. i) Given prices, individual strategies* $(c_{i,t}^f, l_{i,t}^f, a_{i,t}^f)_{i,t\geq 0}^f$ *solve the agent's optimization program in equations (7)–(9). ii) Financial, labor, and goods markets clear: for any* $t \geq 0$*, equations (12) and (13) hold. iii) The government budget is balanced: equation (3) holds for all* $t \geq 0$*. iv) The pre-tax factor prices* $(\tilde{w}_t, \tilde{r}_t)_{t\geq 0}$ *are consistent with the firm's program (1).*

Using the previous definition, we now state the formal definition of the Ramsey equilibrium, and the stationary Ramsey equilibrium.

Definition 2 (Ramsey Equilibrium (RE))**.** *A Ramsey Equilibrium is a competitive equilibrium, which generates the highest welfare, measured by* W_0*, over the set of fiscal policies* $(\tau_t^K, \kappa_t, \tau_t, B_t)_{t\geq 0}$ *satisfying the governmental budget constraint.*

Definition 3 (Stationary Ramsey Equilibrium (SRE))**.** *A stationary Ramsey equilibrium is a Ramsey equilibrium for which aggregate quantities* $(K_t, L_t, Y_t)_{t\geq 0}$*, prices* $(\tilde{w}_t, \tilde{r}_t)_{t\geq 0}$*, fiscal policy* $(\tau_t^K, \kappa_t, \tau_t, B_t)_{t\geq 0}$*, and public spending* $(G_t)_{t\geq 0}$ *are constant.*

First-best allocation. A natural candidate against which to compare the outcome of the Ramsey equilibrium is the first-best allocation. The latter is the solution of the program maximizing aggregate social welfare W_0, subject only to the resource condition. Formally, it solves the following program:

$$\max_{((c_{i,t},l_{i,t})_{i\in I},L_t,K_t)_{t\geq 0}} W_0, \tag{15}$$

$$\text{s.t. } \sum_{f=1}^{F} m^f \int_i c_{i,t}^f \ell^f(di) + G_t + K_t = K_{t-1} + F(K_{t-1}, L_t), \tag{16}$$

$$\text{and } \sum_{f=1}^{F} m^f \int y_{i,t}^f l_{i,t}^f \ell^f(di) = L_t,\ K_{-1} \text{ given.}$$

The solution of this program provides the Pareto frontier of this economy by varying the social weights ω^f.

3 Analyzing Existence and Dynamics in a Simple Model

We first study optimal fiscal policy in a simple model, in which we can derive analytical results. The main simplifying assumption is to consider deterministic productivity fluctuations, as introduced by Woodford (1990). The gain of this approach is that it yields analytical solutions—including a characterization of the Ramsey allocation—but also that it provides the proof that positive optimal capital taxation and public debt are the results of credit constraints, and not of incomplete insurance markets.

The simplifying assumptions introduced in the environment of Section 2 are as follows.

Assumption A.

1. *The labor tax is linear: in (2) we set* $\tau_t = 0$ *and denote* $\tau_t^L := 1-\kappa_t$ *such that* $T_t(\tilde{w}yl) := \tau_t^L \tilde{w}yl$.
2. *The credit constraint is set to zero:* $\underline{a} = 0$.
3. *There is only one productivity process (*$F = 1$ *and* $m_1 = 1$*), which can take only two values:* 0 *and* 1*. The transition matrix is anti-diagonal:* $\Pi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ *and there is initially a unit mass of agents in each productivity level.*

In this setup, the planner is endowed with three instruments: a linear capital tax, a linear labor tax, and public debt. Agents face deterministic productivity variations, which they can smooth using their savings, subject to borrowing limits. Due to the assumption about the initial distribution and the anti-diagonal transition matrix, the total population has been renormalized to 2 and in every period there is a population of mass one that is employed (called "employed", with subscript e) and another that is unemployed (called "unemployed", with subscript u).

The remainder of this section is organized as follows. In Section 3.1, we characterize the planner's first-order conditions for general utility functions, and then discuss different standard cases (CRRA, DRRA, CARA, GHH, KPR, among others). In Section 3.2, we focus on the GHH utility function and derive formal existence conditions. In Section 3.3, we provide our main results about the dynamics of public debt in the simple model. Finally, in Section 3.4, we relax the assumption of a homogeneous population and study the effects of the SWF weights on the equilibrium capital tax.

3.1 Characterizing the Planner's FOCs in an SRE

As a preliminary remark, observe that in any non-trivial equilibrium, employed agents cannot be credit-constrained at any date: otherwise unemployed agents would consume zero, as they do not earn any labor income. Thus, there are only two possible steady-state equilibria: one in which unemployed agents are not credit-constrained, and one in which they are. Thus, the Ramsey program can be written as follows:

$$\max_{(c_{e,t},c_{u,t},a_{e,t},a_{u,t},l_{e,t}B_t,A_t,R_t,w_t)} \sum_{t=0}^{\infty} \beta^t \Big(U(c_{e,t},l_{e,t}) + U(c_{u,t},0) \Big), \tag{17}$$

$$\text{s.t. } c_{e,t} + a_{e,t} = R_t a_{u,t-1} + w_t l_{e,t}, \tag{18}$$

$$c_{u,t} + a_{u,t} = R_t a_{e,t-1}, \tag{19}$$

$$U_c(c_{e,t},l_{e,t}) = \beta R_{t+1} U_c(c_{u,t+1},0), \tag{20}$$

$$U_c(c_{u,t},0) \geq \beta R_{t+1} U_c(c_{e,t+1},l_{e,t+1}), \text{ with equality if } a_{u,t} > 0, \tag{21}$$

$$-U_l(c_{e,t},l_{e,t}) = w_t U_c(c_{e,t},l_{e,t}), \tag{22}$$

$$F(A_{t-1} - B_{t-1}, l_{e,t}) + B_t \geq G_t + B_{t-1} + (R_t - 1)A_{t-1} + w_t l_{e,t}, \tag{23}$$

$$A_t = a_{e,t} + a_{u,t}, \tag{24}$$

$$a_{e,t}, a_{u,t} \geq 0, \tag{25}$$

$$c_{e,t}, c_{u,t} > 0 \text{ and } l_{e,t}, l_{u,t} \geq 0. \tag{26}$$

The planner maximizes the aggregate welfare criterion (17) subject to the following: the constraints (18)–(22), which guarantee the optimality of individual choices (budget constraints, Euler equations, and labor FOC, respectively); the governmental budget constraint (23); the financial market clearing condition (24); the credit constraints (25); and the positivity constraints (26).

SRE with non-binding credit constraints. When credit constraints do not bind, we recover the seminal result of Chamley (1986) and Judd (1985), that the optimal capital tax is zero in an SRE. Indeed, on the one hand, using the two Euler equations (20) and (21), we find $\beta R = 1$. On the other hand, the planner's FOC for public debt implies the modified golden rule at the steady-state: $\beta(1+\tilde{r}) = 1$. We deduce that post- and pre-tax rates must coincide: $R = 1 + \tilde{r}$ and the capital tax is null: $\tau^K = 0$.[8] A positive capital tax at the steady state can only be optimal in an equilibrium with binding credit constraints.

SRE with binding credit constraints. We characterize the equilibrium where the credit constraints bind for unemployed agents ($a_{u,t} = 0$ for all t). Since unemployed agents are credit constrained, the employed and unemployed budget constraints (18) and (19) simplify into $c_{e,t} = w_t l_{e,t} - a_{e,t}$ and $c_{u,t} = R_t a_{e,t-1}$.

---

[8] We here characterize a SRE. Its existence is however not ensured in all cases, as shown by Straub and Werning (2020), and as we discuss below.

To summarize the effects at stake when the planner optimally sets the capital and labor taxes, we first define the following quantities:

$$\sigma_e := -c_e \frac{U_{cc}(c_e, l_e)}{U_c(c_e, l_e)},\ \sigma_u := -c_u \frac{U_{cc}(c_u, 0)}{U_c(c_u, 0)}, \tag{27}$$

$$\varphi_e := \left(l_e \frac{U_{ll}(c_e, l_e)}{U_l(c_e, l_e)} \right)^{-1},\ \varsigma^l_{c,e} := l_e \frac{U_{cl}(c_e, l_e)}{U_c(c_e, l_e)},\ \varsigma^c_{l,e} := c_e \frac{U_{cl}(c_e, l_e)}{U_l(c_e, l_e)}. \tag{28}$$

The quantities $\sigma_e, \sigma_u \geq 0$ in equation (27) are the inverse of the intertemporal elasticity of substitution (IES) for employed and unemployed agents. The inverse of the IES is also, by analogy to the static case, referred to as the relative risk aversion (RRA), even though in our case there is no risk and the correct interpretation is in terms of an elasticity. The IES is constant for CRRA utility functions, and hence identical for both agents independently of consumption and labor choices. The quantity $\varphi_e \geq 0$ is the Frisch elasticity of the labor supply for employed agents. Finally, $\varsigma^l_{c,e}$ is the elasticity of the marginal utility of consumption of employed agents, $U_c(c_e, l_e)$, to labor supply, while $\varsigma^c_{l,e}$ is the elasticity of the marginal utility of labor supply of employed agents, $U_l(c_e, l_e)$, to consumption. These two last terms are null when the utility function U is separable in consumption and labor. The next Proposition first presents a characterization of an SRE.

Proposition 1. *In any standard SRE with a binding credit constraint for unemployed households, we have:*

1. $1 + F_K = \frac{1}{\beta}$*;*

2. *The post-tax interest and wage rates satisfy:*

$$\underbrace{1 - \beta R}_{\textit{Smoothing wedge}} = \underbrace{\frac{F_L - w}{w}}_{\textit{Labor wedge}} \underbrace{\frac{\sigma_u - \sigma_e + \varsigma^l_{c,e}}{\sigma_e + \frac{1}{\psi_e} - \varsigma^l_{c,e} + \varsigma^c_{l,e}}}_{\textit{Net Distributional Gain}}. \tag{29}$$

The proof can be found in Appendix A.6.1.[9] Before commenting on the equality (29), two remarks are in order. The first part of the proposition has been well known since Aiyagari (1995), and is called the modified golden rule. Since the government faces no credit constraint, it is optimal to set the public debt so that the marginal productivity

---

[9] We refer to a standard SRE as an equilibrium where Lagrange multipliers are finite. We characterize the existence of these equilibria in Proposition 3.2.1 below. Lansing (1999) has shown that in the special case of an IES of 1, a SRE could exist in a representative agent model, with diverging Lagrange multipliers. This is not the case in our model as shown in Appendix A.3.8.

of capital equals the discount rate. As noted above, this depends solely on the planner's ability to adjust public debt freely, and is independent of whether or not credit constraints are binding for private agents. A second remark is that condition (29) is a necessary condition of the SRE when credit constraints are binding for unemployed agents. This is a first step towards characterizing the existence of such equilibria, which is not ensured at this stage.

To better understand the naming of terms and the intuitions in equation (29), it is worth considering the planner's optimality conditions. Different perspectives on the planner's optimization program are possible; however, the simplest way to understand the relationship (29) between post-tax interest and wage rates is to think of the planner as jointly setting savings and the labor supply, while internalizing the general equilibrium effects (through prices) of its choices. First, the FOC associated with the savings choice can be written as:

$$1 - \beta R = \Xi \times \left(\sigma_u - \sigma_e + \varsigma^l_{c,e}\right), \tag{30}$$

where $\Xi \geq 0$ is a product of various Lagrange multipliers summarizing equilibrium distortions. When the planner sets the level of savings, equation (30) shows that the planner trades off the marginal benefit of higher savings, implying a larger tax base (on the left hand side), with the related marginal cost that channels through higher post-tax interest and wage rates (on the right hand side). The marginal benefit is proportional to the capital tax, since an extra unit of saving increases the capital tax base. Indeed, because of the modified golden rule, we have: $1 - \beta R = \beta(1 + F_K - R) = \beta(\tilde{r} - r) = (1 - \beta)\tau^K$. When the capital tax is positive, $\beta R < 1$ and the Euler equation implies imperfect consumption smoothing. Thus, the term $1 - \beta R$ is called a "smoothing wedge." Absent any costs, the planner would set savings so as to maximize the benefit for its resources – and hence set the marginal benefit to zero. This would imply a zero capital tax and $1 + F_K = R$, as discussed after equation (6).

The planner does not maximize the benefit of savings for its resources because of the externality of savings that tends to increase post-tax interest and wage rates, which is detrimental for the planner's resources as explained after equation (6). Note that there is no direct welfare effect of this extra saving, due to the envelope theorem. By the construction of the Ramsey program, the planner chooses savings that are optimal for households and thus consistent with the Euler equation. The externality of savings on prices channels through the agents' consumption. Extra savings raise the consumption of the unemployed and reduce the consumption of employed. Both effects contribute to raise the post-tax interest rate via the Euler equation (20) of employed agents, because unemployed agents are credit-constrained. However, the lower consumption of the employed in general decreases the wage rate due to the FOC on the labor supply (22).[10] When the net total effect of extra savings on interest and wage rates is detrimental to the planner's resources, the externality of savings on factor prices is a cost for the planner and limits the increase in savings to value that means positive capital tax.

When the utility function is separable, the externality of savings on post-tax factor prices is proportional to the curvature of the utility function, and more precisely to the inverse of the IES. For unemployed agents, since their budget constraint is $R_{t+1}a_{e,t} = c_{u,t+1}$, the effect of a change in prices due to a marginal increase in savings (and thus higher consumption by unemployed agents) is proportional to $\frac{\partial R_{t+1}}{\partial c_{u,t+1}} a_{e,t}$, which is equal to $\frac{\partial \log R_{t+1}}{\partial \log c_{u,t+1}}$. Because of the Euler equation, this is the inverse of the intertemporal elasticity of substitution, σ_u. For employed agents, the extra savings (and hence the lower consumption of the employed) affect both the interest and the wage rates and yields the sum of elasticities $-a_{e,t}\frac{\partial \log R_{t+1}}{\partial c_{e,t}} - w_t l_{e,t}\frac{\partial \log w_t}{\partial c_{e,t}}$, which using the budget constraint (18), the Euler equation (20), and the FOC (22) on the labor supply becomes $-\sigma_e$. Therefore, in the separable case, the externality of savings on factor prices is proportional to the gap between the inverse IES of employed and unemployed agents. In the general non-separable case, the effect on the employed further includes a cross-derivative term, $\varsigma^l_{c,e}$ coming from the term $\frac{\partial \log w_t}{\partial c_{e,t}}$.

As a take-away, equation (30) states that when setting aggregate savings, the planner trades off the benefits of a larger capital tax base with the cost of higher post-tax factor prices. An alternative interpretation is that at the equilibrium, the planner uses the capital tax to correct the externality of savings on factor prices that would otherwise let the agents save "too much" from a social perspective: the planner uses their instruments to correct the negative externality of post-tax factor prices on savings.[11]

Additionally, the FOC related to the labor supply of the employed can be written as:

$$\frac{F_L - w}{w} = \Xi \times \left(\sigma_e + \frac{1}{\psi_e} - \varsigma^l_{c,e} + \varsigma^c_{l,e}\right), \tag{31}$$

---

[10] The effect actually depends on the cross derivative of the utility function with respect to consumption and labor. When the utility is separable, a lower consumption of employed agents always decreases the wage rate. When $U_{cl} < 0$, the effect is mitigated and can possibly have a different sign. For instance, with a GHH utility function, the wage rate is independent of employed consumption.

[11] This argument is reminiscent of the one in Dávila et al. (2012), where in the laissez-faire economy agents can save too much because they do internalize the effect of their savings on factor prices. In our environment, at the SRE the planner considers the effect of the price externality on the budget constraint of the government.

with the same multiplier $\Xi \geq 0$ as in FOC (30). The interpretation follows the same line as the interpretation of FOC (30). The planner trades off the benefit of an additional hour of labor supply in terms of a higher labor tax base, with the cost of higher post-tax factor prices. On the one hand, for the planner's resources (6), an additional hour of labor supply increases total output by $F_{L,t}$ at the cost of higher (after-tax) wage w_t. Thus, the effect is proportional to the gap between $F_{L,t}$ and w_t, and hence to the labor tax. An equivalent view is that the higher labor supply increases the base of the labor tax, and thus the planner's resources, in proportion to τ_t^L. On the other hand, the cost again operates through the externality of the labor supply on post-tax factor prices – and hence only concerns employed agents. Since the extra labor supply also increases consumption of the employed, the effect in the separable case is proportional to $\frac{\partial \log w_t}{\partial \log c_{e,t}} + \frac{\partial \log w_t}{\partial \log l_{e,t}}$, which equals $\sigma_e + 1/\varphi_e$. The non-separable case features two additional interaction terms, equal to $-\varsigma_{c,e}^{l} + \varsigma_{l,e}^{c}$ involving cross derivatives of the utility function.

Separable utility functions : CRRA, DRRA, CARA and IRRA cases. The equality (29) has important implications for specific utility functions. For separable utility functions, the cross-derivative terms are zero and $\varsigma_{c,e}^{l} = \varsigma_{l,e}^{c} = 0$, which simplifies the algebra (as well as the intuition, somewhat). We distinguish three cases: Constant, Decreasing and Increasing Relative Risk Aversion utility functions (CRRA, DRRA, IRRA respectively). Detailed calculations and numerical examples can be found in Appendix A.2.

First, the CRRA separable case, $U(c,l) = \frac{c^{1-\sigma}-1}{1-\sigma} - v(l)$, with $\sigma \neq 1$, or $U(c,l) = \log c - v(l)$, implies $\sigma_u = \sigma_e = \sigma$ independently of consumption levels. Since there is no externality of savings on prices, the planner sets savings so as to maximize their impacts on its resources: $1 + F_K = R$, or $\tau^K = 0$. Steady-state capital taxes are null in this equilibrium. This outcome generalizes the result of Chamley (1986) to an economy with an occasionally-binding credit constraint, but only for this specific utility function. It is consistent with the claims of Chen et al. (2020); Auclert et al. (2022); Chien and Wen (2024) – the latter provided a general proof considering the CRRA case. We summarize this result in the next corollary.

Corollary 1. *If the utility function is $U(c,l) = u(c) - v(l)$, where u is CRRA, then the capital tax is* 0 *in SRE.*

For other cases, observe that a positive capital tax implies $\beta R < 1$ and hence $c_u < c_e$ when credit constraints are binding. For DRRA functions, we have $\sigma_u > \sigma_e$ and the net distributional gains in (29) are positive. Savings imply a negative externality on post-tax factor prices. The planner avoids to increase these factor prices too much and ends up with a positive capital tax. Thus, the equilibrium, if it exists, will then feature positive capital and labor taxes. We provide examples of such equilibria for Stone-Geary utility functions in Appendix A.2.1.1 and for Fishburn utility functions in Appendix A.2.1.2.[12]

For the IRRA utility function, the situation is slightly more involved. On the one hand, savings still involve a negative externality on factor. However, since $\sigma_u < \sigma_e$, the negative externality of savings in (30) is the combination of savings *decreasing* interest rates and wages (rather than increasing them as in the DRRA case), and of post-tax factor prices having a *positive* externality on savings (and not negative as in the usual case discussed after equation (30)). The latter relationship implies that the labor supply in (30) has a positive externality on agents, and hence that labor should be subsidized. Therefore, the equilibrium, if it exists, features a positive capital tax but a *negative* labor tax. A standard example of an IRRA utility function is a Constant Absolute Risk Aversion (CARA) utility function, $U(c,l) = -\frac{1}{\gamma}e^{-\gamma c} - \frac{1}{\chi\varphi}e^{\varphi l}$, where $\gamma, \varphi > 0$. These functions are used by Acharya and Dogra (2021) and Acharya et al. (2023), among others. We have $\sigma(c) = \gamma c$, which is increasing. See Appendix A.2.2 for a numerical example with a CARA utility function.

Non-Separable utility functions: The GHH case. A standard non-separable utility function considered in the literature is the GHH utility function. This instantaneous utility function U is:

$$U(c,l) := u\left(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}\right), \tag{32}$$

where $\varphi > 0$ is the Frisch elasticity of labor supply, $\chi > 0$ scales labor disutility, and the function u has a constant IES equal to $1/\sigma \geq 0$. This function has the property that the labor supply exhibits no wealth effect. It has been used for instance in the seminal contributions of Aiyagari (1995) and Diamond (1998) to obtain analytical results, but also in some quantitative work (e.g., Bayer et al., 2019 and Açikgöz et al., 2022), as it simplifies the computation of the equilibrium allocation.

Applying the equality (29) in the context of the GHH utility function yields the following relationships between smoothing and labor wedges:

$$1 - \beta R = \frac{F_L - w}{w}\varphi\sigma\left(1 + \beta\left(\beta R\right)^{\frac{1}{\sigma}-1}\right). \tag{33}$$

---

[12] Fishburn (1977) analyzes a utility function which is isoelastic below a threshold and linear after it. This utility function was used by Challe and Ragot (2016) and LeGrand and Ragot (2018) because it generates tractable models.

In the case where $\sigma = 1$ (and thus $u(c) := \log(c)$), this expression can be further simplified into a simple relationship relating the capital and labor taxes:

$$(1-\beta)\tau^K = \frac{\tau^L}{1-\tau^L}\varphi(1+\beta). \tag{34}$$

This shows that in equilibrium, the capital tax increases with the labor tax: both distortions increase together with the financial requirements that the planner has to finance.[13] In particular, the capital tax is positive whenever the labor tax is.[14]

Finally, for the sake of completeness, we also consider another example of a non-separable utility function, which is the one of King-Plosser-Rebelo (King et al., 1988), used for example by Dyrda and Pedroni (2022). We provide results in Appendix A.2.3. We find that an SRE with positive capital and labor taxes can exist only for some restrictions on the parameters, which is the case considered by Dyrda and Pedroni (2022).

3.2 An Existence Result in the GHH Case

We now focus on the GHH case with an IES of 1. We consider this utility function for the rest of the paper.

Before presenting these existence conditions, three remarks are in order. First, even in this simple framework, we must check that the Karush–Kuhn–Tucker conditions apply to our problem, and that the FOCs actually characterize an optimum. Because of the nonlinearity of the constraints (20)–(23), the standard Slater (1950) condition does not apply in our optimization program. Therefore, we must check another constraint qualification; this is done in Appendix A.3.2, where we verify that the linear independence constraint qualification holds. Second, we verify that the second-order conditions of the Ramsey planner are also fulfilled, such that the FOCs indeed characterize a maximum. This is done in Appendix A.3.3. Finally, we also consider an IES different from 1 in Appendix A.3.8, but we keep the simplest case in this Section.

3.2.1 First-Best Allocation and Possible Decentralization

As is standard in this type of problem, the first-best outcome can be attained if public spending is not too high. In this case, public debt is negative (the government thus holds

---

[13] One can check that τ^K/τ^L increases with the discount factor β and the Frisch elasticity.

[14] When β increases towards 1 from below, equation (34) would imply that the capital tax would increase without limit relative to the labor tax. However, the equilibrium does not exist in this case, as shown in Section 3.2 below, where the existence proof is provided.

a share of the capital stock) and the government finances public spending out of interest payments on its asset holdings. This is stated formally in the next proposition, whose proof can be found in Appendix A.3.1, together with the value of the steady-state first-best level of output Y_{FB}.[15]

Proposition 2. *Define:*

$$\overline{g}_1 := \frac{1-\beta}{\beta}\frac{\alpha}{1/\beta+\delta-1} - \frac{1-\beta}{1+\beta}\frac{1-\alpha}{\varphi+1}. \tag{35}$$

If public spending verifies $G \leq \overline{g}_1 Y_{FB}$, then the steady-state Ramsey allocation is the first-best steady-state allocation characterized by zero taxes and perfect consumption smoothing.

When $G > \overline{g}_1 Y_{FB}$, the first-best allocation cannot be sustained, because financing such a large public spending out of capital income requires the government to hold a financial asset position that would exceed the total capital stock in the economy[16].

3.2.2 Binding Credit Constraints

When the first-best allocation cannot be sustained, the credit constraints of the unemployed agents must bind. The next proposition characterizes the existence of the equilibrium with credit constraints.

Proposition 3. *There exist two thresholds $\overline{g}_{La}$ and $\overline{g}_{SW}$, defined as:*

$$\overline{g}_{La} := \frac{1-\alpha}{\varphi}\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}\right)(1-\overline{\tau}^L_{La})^{1+\varphi}, \tag{36}$$

$$\textit{where: } \overline{\tau}^L_{La} = \frac{1}{1+\varphi} - \frac{1}{1-\alpha}\frac{\varphi}{1+\varphi}\frac{\overline{g}_1}{1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}}, \tag{37}$$

$$\overline{g}_{SW} := \overline{g}_1 + (1-\alpha)\left(1+\frac{1-\beta}{1+\beta}\frac{1}{1+\varphi}+\frac{\varphi}{1+\varphi}\right)(1-\frac{1}{1+\varphi(1+\beta)})^{\varphi}, \tag{38}$$

such that when $\overline{g}_1 Y_{FB} < G \leq \min(\overline{g}_{SW}, \overline{g}_{La}) \times Y_{FB}$, there exists a unique SRE with a binding credit constraint for unemployed agents, where both taxes τ^L and τ^K are positive.

The proposition is proved in Appendices A.3.4 and A.3.5. In addition to the non-first-best condition, $\overline{g}_1 Y_{FB} < G$, the existence of the steady-state equilibrium is subject to two

---

[15] This non first-best condition is the condition identified in more general settings by Albanesi and Armenter (2012), for the optimal steady-state capital tax not to be zero.

[16] This would imply that the government lends resources to households, which is prevented by credit constraints.

additional conditions, reflected in the two thresholds ($\overline{g}_{SW}$ and $\overline{g}_{La}$) for public spending. The first threshold $\overline{g}_{SW}$ ensures that the consumption of unemployed agents is positive and that the Lagrange multiplier μ on the governmental budget constraint (23) is constant and finite.[17] When G increases toward $\overline{g}_{SW} Y_{FB}$, the planner needs to raise capital taxes so high that the post-tax return of savings tends to zero, as does the consumption of unemployed agents. In this case, the government finds it infinitely costly to implement the steady-state optimal allocation, as taxes and distortions become infinitely high. This explains why the Lagrange multiplier on the governmental budget, μ, tends to infinity. At the threshold $\overline{g}_{SW} Y_{FB}$, the planner prefers to switch to a non-stationary equilibrium, where output is decreasing. This limit case has been discussed recently by Straub and Werning (2020), justifying the SW subscript and the denomination of Straub-Werning condition. To summarize, if $G > \overline{g}_{SW} Y_{FB}$, then no (stationary) steady-state equilibrium exists, and a non-stationary equilibrium may exist, as studied in Appendix A.3.6.

The threshold $\overline{g}_{La}$ corresponds to a more traditional Laffer condition. When G is higher than this last threshold, not enough resources can be raised in the economy through the distorting taxes to finance public spending. We prove in Appendix A.3.4 that the constraints $\overline{g}_1 Y_{FB} < G \leq \min(\overline{g}_{SW}, \overline{g}_{La}) \times Y_{FB}$ are compatible for some G and some parameter values. However, stating which of $\overline{g}_{SW}$ or $\overline{g}_{La}$ is greater in general is not possible, as both cases are possible depending on parameter specification. For instance, when α is close to 1, we have $\overline{g}_{SW} < \overline{g}_{La}$, while when both α and φ are close to 0, the opposite holds.

The allocation can be derived explicitly in this tractable economy. Because of the GHH assumption, the labor supply of the employed agent is given by $l_e = (\chi w)^{\varphi}$, while their saving is $a_e = \frac{\beta}{1+\beta} \frac{w(\chi w)^{\varphi}}{1+\varphi}$. Using the expression (34) together with the government's budget constraint (23) at steady state, we can obtain an explicit expression for the post-tax real wage, thus providing an analytical solution to the allocation: $w = \frac{(F(k_{FB},1)-G)(1+\varphi)+w_{FB}\varphi}{1+2\varphi+\frac{1-\beta}{1+\beta}}$, where $w_{FB} = (1-\alpha)k_{FB}^{\alpha}$ and $k_{FB} = \left(\frac{\alpha}{\frac{1}{\beta}+\delta-1}\right)^{\frac{1}{1-\alpha}}$ are the first-best capital-to-labor ratio and wage rate, respectively (see Appendix A.3 for further details). The labor tax is then $\tau^L = 1 - w/w_{FB}$ and the capital tax is given by equation (34). Finally, public debt can be deduced

---

[17] In the case of the log-GHH utility function, the two conditions are identical. When the IES differs from one, the conditions $c_u > 0$ and $\mu > 0$ differ from each other, and the condition $\mu > 0$ typically binds first. See Appendix A.3.8 for further details. In addition we also show that the allocation and the dynamic of Lagrange multipliers are continuous in the IES, such that the specific case identified by Lansing (1999) and Reinhorn (2019) (i.e. diverging multipliers and converging allocation) for the IES of 1 does not exist in our environment.

from the capital market clearing condition: $B = a_e - K = (\chi w)^{\varphi}(\frac{\beta}{1+\beta}\frac{1-\tau^L}{1+\varphi}w_{FB} - k_{FB})$.[18]

3.2.3 When Is Optimal Public Debt Positive?

This model can generate a positive amount of optimal public debt, as stated in the following result, which is proved in Appendix A.3.7.

Result 1. *There exists a threshold $\overline{g}_{pos}$ defined as:*

$$\overline{g}_{pos} = \frac{1+\beta}{1-\beta}(1+2\varphi)(-\overline{g}_1), \tag{39}$$

such that steady-state public debt is positive, $B \geq 0$, iff $\overline{g}_1 \leq 0$ and $G \leq \overline{g}_{pos} Y_{FB}$.

The joint positivity of public debt and capital tax is not obvious: why would the planner provide more public debt to the market (more liquidity in the sense of Woodford, 1990) and then tax the return on public debt with a positive capital tax? In an equilibrium with positive public debt, the equilibrium savings of employed agents are higher than the optimal capital stock, and the extra savings are absorbed by the public debt. From this allocation, decreasing public debt would inefficiently increase the capital stock, and would further require an increase in the capital tax to reduce savings, which would hinder consumption smoothing. Thus, public debt enables the planner to absorb the excess of savings and reconcile the high savings of private agents with the optimal capital stock without affecting consumption smoothing too drastically. This explains the condition $\overline{g}_1 \leq 0$, which states that, no matter the level of public spending, it is never optimal for the government to hold a share of capital to finance public spending. The agents' savings motives are indeed too strong given the level of capital in the economy. The second condition $G \leq \overline{g}_{\text{pos}} Y_{FB}$ comes from the fact that a high level of public spending requires a high level of distorting taxes, and thus a lower level of private saving. As a consequence, the public debt necessary to absorb the excess saving is decreasing with G. If G is too high, optimal debt becomes negative.

3.2.4 Conclusion About Existence

We have characterized four conditions, given by $\overline{g}_1, \overline{g}_{SW}, \overline{g}_{La}$ and $\overline{g}_{\text{pos}}$ for the existence of an SRE with positive optimal capital taxation and public debt. These conditions

---

[18] When an SRE with a positive capital tax exists, we can show that the planner does not want to implement any lump-sum transfers, since they would require raising distortionary taxes to implement. The proof is in Appendix A.3.4.

are independent, but can be satisfied simultaneously for some parameter values.[19] For instance, we use the following parameters (ensuring existence) to study the steady state and dynamics of the simple model in Appendix A.4.3. The parameters are $\alpha = 0.3, \beta = 0.7, \varphi = 0.3, \delta = 1, G = 0.01, \chi = 1$, and one can check that $\bar{g}_1 Y_{FB} < G$, $G \leq \bar{g}_{SW} Y_{FB}$, $G < \bar{g}_{La} Y_{FB}$, and $G < \bar{g}_{pos} Y_{FB}$. This economy has an equilibrium capital tax of 6%, a labor tax of 3%, and a (small) positive public debt. Larger values of public spending G reduce the public debt, which can become negative. There is actually no interior steady state, as discussed in Appendix A.3.5, although non-interior equilibria may exist. Since these properties are close to those derived by Straub and Werning (2020), we present them in Appendix A.3.6.

3.3 Dynamic Analysis of Public Debt

We now use the simple model to derive some insights about the optimal dynamics of public debt after a public spending shock. We assume full capital depreciation, $\delta = 1$, and consider a first-order approximation of the model.

Time consistency. It is interesting to note that in the log-GHH case with log period utility (32), the program of the planner is time-consistent, although capital is fixed in period 0 and capital taxes are chosen at period 0 (which is not the case in more general settings, as discussed below and in LeGrand and Ragot, 2023). Indeed, in this case, and when credit constraints bind, the saving of employed agents does not depend on the post-tax real interest rate, but only on the post-tax real wage (see Appendix A.3.4).[20]

Linearization. We denote with a hat the relative deviation of a variable from its steady-state value: $\widehat{x}_t = \frac{x_t - x}{x}$ for a generic variable x_t with steady-state value x. The public spending shock is assumed to be defined as follows:

$$\widehat{G}_t = \begin{cases} \widehat{G}_0 & \text{if } t = 0, \\ \rho_G \widehat{G}_{t-1} & \text{if } t > 0, \end{cases} \tag{40}$$

---

[19] Regarding uniqueness, in the log-GHH case, we can prove the uniqueness of the SRE. However, this is not true in the general case, when the IES is different from one. Even with a GHH utility function, if the IES differs from one, multiple allocations can satisfy the planner's FOCs while satisfying the Straub-Werning and Laffer conditions. See Appendix A.3.8 for a numerical example.

[20] More precisely, the Lagrange multipliers on the previous period's Euler equations do not affect the current period allocation. See Appendix A.3.4.

with $\widehat{G}_0$ small enough for a first-order approximation of the dynamics to be relevant, and $\rho_G \in (-1, 1)$. The shock only happens at date $t = 0$ and then persists with parameter ρ_G, as is consistent with our assumption of an MIT shock.

Characterization of the system stability. Our first result is to characterize the stability of the dynamic system that yields the Ramsey allocation, using the FOCs of the planner. Interestingly, the dynamic of the Ramsey allocation can be fully summarized by taking capital as the unique state variable, together with the public spending shock.

Result 2. *The optimal dynamic of the capital stock is given by the following system:*

$$\widehat{K}_t = \rho_K \widehat{K}_{t-1} + \sigma_K \widehat{G}_t, \tag{41}$$

where the coefficients $\rho_K > 0, \sigma_K < 0$*.* ρ_K *does not depend on* ρ_G *and* $\frac{\partial \sigma_K}{\partial \rho_G} > 0$*.*

See Appendix A.4.1 for the expressions of the coefficients and computations. Thus at impact, an increase in public spending diminishes capital, and the higher the persistence of the public spending shock, the stronger the effect.

The dynamic system (41) is stable when the auto-regressive coefficient ρ_K is smaller than one in absolute value. In our setup, this is equivalent to verifying the Blanchard–Kahn conditions. The result regarding system stability is summarized in the following proposition.

Proposition 4. *The system (41) is stable, i.e.,* $|\rho_K| < 1$*, iff*

$$\alpha \leq \frac{1}{1 + (1-\beta)(1+\varphi)}. \tag{42}$$

The dynamic system is stable under (42), which is called the Blanchard-Kahn condition and which imposes an upper bound on α. Note that this upper bound is always strictly smaller than one and hence can be binding. Condition (42) on α always holds when public debt is positive, i.e., when $\overline{g}_1 < 0$. When the capital share α and the Frisch elasticity φ are both high (such that (42) is not fulfilled), a small shock in public spending induces too large a change in the resources of the planner, such that capital diverges.

By induction, we can derive from (40) and (41) the closed-form expression of the impulse response function for optimal capital:

$$\widehat{K}_t = \sigma_K \widehat{G}_0 \frac{\rho_K^{t+1} - \rho_G^{t+1}}{\rho_K - \rho_G}. \tag{43}$$

This allows us to completely characterize the capital path following a public spending shock. At impact and after a positive shock ($\widehat{G}_0 > 0$), the relative variation of capital is always negative by a quantity $\sigma_K \widehat{G}_0 < 0$. Then, the profile of the capital variation is hump-shaped: it starts decreasing further, before increasing and reverting back to zero (see Appendix A.4.2 for further characterization of the dynamics of the capital stock).

Role of the persistence of the public spending shock ρ_G on public debt. From the expression for capital (43), it is possible to derive an explicit expression for the optimal dynamics of public debt:

$$\widehat{B}_t \quad = \quad \widehat{G}_0(\Theta^K \rho_K^t - \Theta^G \rho_G^t). \tag{44}$$

The coefficients Θ^K, Θ^G are functions of the parameters of the model but not of $\widehat{G}_0$ and are provided in equations (A.62) and (A.63) of Appendix A.4.2. These parameters can be either positive or negative. As a consequence, on impact, the change in public debt, $\widehat{B}_0 = \widehat{G}_0(\Theta^K - \Theta^G)$, after a positive public spending shock ($\widehat{G}_0 > 0$) can be either positive or negative, because the sign $\Theta^K - \Theta^G$ is ambiguous. We can characterize the effect of the persistence of the shock on the initial change of public debt, considering two cases. First, we analyze the effect of ρ_G with fixed $\widehat{G}_0$ to understand the mechanisms. Our second experiment focuses on studying the effect of ρ_G while keeping the NPV of public spending unchanged. More formally, we keep the following quantity unchanged, denoted by $N\hat{P}V_0$:

$$N\hat{P}V_0 = \sum_{t=0}^{\infty} \frac{\widehat{G}_t}{R^t} = \widehat{G}_0 \sum_{t=0}^{\infty} \left(\frac{\rho_G}{R}\right)^t = \widehat{G}_0 \frac{R}{R - \rho_G}.$$

Keeping the NPV unchanged while changing ρ_G implies setting the initial size of the shock to $\widehat{G}_0(\rho_G) = N\hat{P}V_0 \frac{R-\rho_G}{R}$. This is summarized in the following proposition.

Proposition 5. *Assume that the steady-state public debt is positive: $B > 0$. Denoting by $\widehat{B}_0$ the variation of public debt on impact, we have*

$$\left.\frac{\partial \widehat{B}_0}{\partial \rho_G}\right|_{\widehat{G}_0} < 0.$$

Moreover, if we further assume $\widehat{B}_0 > 0$, we also have

$$\left.\frac{\partial \widehat{B}_0}{\partial \rho_G}\right|_{N\hat{P}V_0} < 0.$$

See Appendix A.4.2 for the proof. The intuition for why the dynamic of the debt depends on the persistence of the shock is the following. After a positive public spending shock, capital is always falling, but to implement consumption smoothing the planner does not want to decrease private savings (which are used by unemployed agents to consume). Consequently, when the persistence of the shock is low, the planner increases public debt to provide a store of value to private agents. Then, a small increase in future taxes allows one to reduce public debt. When the persistence is high, this strategy is very costly in terms of welfare, because the fall of the capital stock is persistent, and the planner would have to increase taxes to reduce public debt in periods when capital and output are low. Consequently the planner does not increase public debt, in order to avoid having to raise taxes in the future to stabilize this debt. Finally, we check in Appendix A.4.3 with a simple numerical example that Proposition 5 still holds when we consider a non-marginal variation in the persistence.

3.4 Ex-Ante Heterogeneous Populations and Social Weights

We extend the previous GHH case, introducing further heterogeneity between agents, in order to analyze the role of social weights in the determination of the optimal capital tax. We now consider two types of agents facing different labor income risk ($F = 2$ using the notation from Section 2). We consider an environment where one type of agent always remains employed, and will not save in equilibrium (reproducing the environment of Judd, 1985), whereas the other type of agent alternates between employment and unemployment, similarly to the agent in Section 3.1. We use the superscript A to denote the type that alternates between employment and unemployment, and the superscript B for the agents who remain employed, and who will not save in equilibrium. The productivity of employed agents is denoted y^A and y^B. The population size of each type is denoted by $\Omega^x \in [0,1]$, $x = A, B$ with $\Omega^A + \Omega^B = 1$. The planner deviates from a Utilitarian objective, and the social welfare weights on the two types of agents (may) differ from their actual shares in the population. The weights in the social welfare function are denoted by $\omega^x \in [0,1]$, $x = A, B$ with $\omega^A + \omega^B = 1$.

The market clearing condition implies $L_t = \Omega^A y^A l^A_{e,t} + \Omega^B y^B l^B_{e,t}$ for the labor market and $A_t = \Omega^A a^A_{e,t}$ for the capital market. The planner's objective can be written as:

$$\omega^A \sum_{t=0}^{\infty} \beta^t \left(\log(c^A_{e,t} - \chi^{-1} \frac{l_{e,t}^{A,1+1/\varphi}}{1+1/\varphi}) + \log(c^A_{u,t}) \right) + \omega^B \sum_{t=0}^{\infty} \beta^t \log(c^B_{e,t} - \chi^{-1} \frac{l_{e,t}^{B,1+1/\varphi}}{1+1/\varphi}). \quad (45)$$

where we restrict to the log case as before. For the sake of simplicity, we define:

$$\Lambda := \frac{\Omega^B w y^B l_e^B}{\Omega^A w y^A l_e^A} = \frac{\Omega^B (y^B)^{\varphi+1}}{\Omega^A (y^A)^{\varphi+1}},$$

which is the ratio between labor income of type B agents and that of type A agents. It captures the inequality in labor income between the two populations. The further away Λ is from 1, the greater the inequality. The following proposition summarizes our main result.

Proposition 6. *In any interior SRE, the smoothing and labor wedges verify:*

$$\underbrace{\frac{1-\beta R}{\omega^A}}_{\textit{Smoothing wedge}} = \underbrace{\frac{\omega^B}{\omega^A} - (1+\beta)\Lambda}_{\textit{Redistribution}} + \underbrace{\frac{F_L - w}{w}}_{\textit{Labor wedge}} \varphi \underbrace{(1+\beta)(1+\Lambda)}_{\textit{Distributional Gain}}, \tag{46}$$

or equivalently, capital and labor taxes are related by

$$(1-\beta)\frac{\tau^K}{\omega^A} = \frac{\omega^B}{\omega^A} - (1+\beta)\Lambda + \varphi(1+\beta)(1+\Lambda)\frac{\tau^L}{1-\tau^L}. \tag{47}$$

The proof can be found in Appendix A.5. Proposition 6 shows that social preferences (ω^A, ω^B) and the optimal tax system are intertwined. On the one hand, from a given set of social preferences (ω^A, ω^B), the fiscal policy (τ^K, τ^L, B) and the SRE can be derived. In fact, equation (47) gives τ^K as function of τ^L, which is then pinned down by the government budget constraint. Public debt is given by the financial market clearing condition. On the other hand, for a given fiscal policy (τ^K, τ^L, B) and an SRE equilibrium, the social preferences (ω^A, ω^B) of the planner can be derived from equation (47). The latter relationship is known as the *inverse optimal approach*, where the social weights are found in order to replicate an observed fiscal system, which is assumed to be optimal. This approach is used in the general model of Section 4, where we characterize the weights that allow the model's steady state to replicate the actual US fiscal system, in order to study its dynamics.

What is the intuition of equation (46)? Since only type-A agents save, while both types supply labor, the distortions associated with the interest rate affect only type-A agents while both types are affected by the distortions associated with the wage rate. This explains why the smoothing wedge is divided by the weight of type-A agents, ω^A, while the labor wedge is actually divided by the weight of the total population $\omega^A + \omega^B = 1$. The relationship (46) includes a third term called "redistribution", which comes from the fact that social weights of agents differ from their *no-distribution weights.* These latter weights are equal to the inverse of marginal utility, such that the planner would not want to implement any redistribution between types. In our log-GHH setup, $(1+\beta)\Lambda$ is equal to the ratio of type-A marginal utilities to type-B marginal utilities, and thus to the ratio of no-distribution weights. The further the social weights are from the no-distribution weights, the higher the capital tax is relative to the labor tax. This is consistent with the fact that type-B agents pay only the labor tax (but not the capital tax), so increasing their social weight increases the social welfare impact of higher labor taxes. The effect of the weight on the redistribution term always dominates its effect on the smoothing wedge term. Thus, social weights play an intuitive role in the composition of the tax scheme: A higher weight ω^B on type B agents increases the capital tax relative to the labor tax. Finally, we can observe that the effect of a higher actual share Ω^B of type B agents on the capital tax (i.e., a higher Λ) is ambiguous, as can be seen in equation (47). On the one hand, it increases the total labor income in the economy (since B-agents are always employed), which increases the tax base of the labor tax, allowing a reduction of the labor tax (for a given capital tax). On the other hand, a higher share Ω^B reduces the redistribution motive as it reduces the gap between relative social weights and relative no-distribution weights. This tends to raise the labor tax relative to the capital tax.

4 The General Model

We now show that the previous results about the price externality and the optimal public debt dynamics hold in the general model. We now analyze the model of Section 2, while considering a GHH utility function.

4.1 Description and Planner's FOCs

Taking advantage of the GHH utility function allows us to simplify the Ramsey program, with some changes of variables. First, the labor choice of an agent i of type f, or an (i,f)-agent in short, given by (11) can be written as $l_{i,t}^f = l_t (y_{i,t}^f)^{\frac{1-\tau_t}{1/\varphi+\tau_t}}$, where:

$$l_t := (\chi(1-\tau_t)w_t)^{\frac{1}{1/\varphi+\tau_t}}. \tag{48}$$

The quantity l_t can be interpreted as the labor supply of an agent endowed with a productivity of 1, and will hence be called the *unitary labor supply*.[21] Second, we define

---

[21] This change in variable makes it unnecessary to solve for the labor supply of individual agents.

an increasing transformation of the progressivity τ_t as

$$\tilde{\tau}_t := \frac{(1/\varphi + 1)(1 - \tau_t)}{1/\varphi + \tau_t} \in (0, 1 + \varphi], \tag{49}$$

where $\tilde{\tau}_t = 1 + \varphi$ corresponds to linear taxation and $\tilde{\tau}_t \to 0$ to full income redistribution (i.e., $\tau_t = 1$, which is always a dominated option for the planner). Third, we define the aggregate quantity $x_{i,t}^f := c_{i,t}^f - \chi^{-1} \frac{(l_{i,t}^f)^{1+1/\varphi}}{1+1/\varphi}$, such that the period utility of an agent (i, f) is simply denoted $u(x_{i,t}^f)$, while their budget constraint is:

$$x_{i,t}^f = (1 + r_t) a_{i,t-1}^f - a_{i,t}^f + \frac{1}{\chi \tilde{\tau}_t} l_t^{1/\varphi+1} (y_{i,t}^f)^{\tilde{\tau}_t}.$$

We can similarly rewrite the governmental budget constraint (6) using this notation. Formally, the Ramsey program can be written as follows:[22]

$$\max_{\left(r_t, \tilde{\tau}_t, B_t, K_t, L_t, l_t, ((a_{i,t}^f, x_{i,t}^f, \nu_{i,t}^f)_{i \in \mathcal{I}^f})_{f \in \{1,\ldots,F\}}\right)_{t \geq 0}} \sum_{f=1}^{F} m^f \omega^f \sum_{t=0}^{\infty} \beta^t \int_i u(x_{i,t}^f) \ell^f(di), \tag{50}$$

$$\begin{aligned} G_t + T_t + r_t A_{t-1} + \left(\frac{1}{\tilde{\tau}_t} + \frac{1}{1/\varphi + 1} \right) \frac{l_t^{1/\varphi+1}}{\chi} \sum_{f=1}^{F} m^f \int_i (y_{i,t}^f)^{\tilde{\tau}_t} \ell(di) = \\ F(A_{t-1} - B_{t-1}, L_t) + B_t - B_{t-1}, \end{aligned} \tag{51}$$

$$\text{for all } i, f\colon\ x_{i,t}^f = (1 + r_t) a_{i,t-1}^f - a_{i,t}^f + \frac{1}{\chi \tilde{\tau}_t} l_t^{1/\varphi+1} (y_{i,t}^f)^{\tilde{\tau}_t}, \tag{52}$$

$$u'(x_{i,t}^f) = \beta \mathbb{E}_t[(1 + r_{t+1}) u'(x_{i,t+1}^f)] + \nu_{i,t}^f, \tag{53}$$

$$a_{i,t}^f \geq -\bar{a},\ \nu_{i,t}^f (a_{i,t}^f + \bar{a}) = 0,\ \nu_{i,t}^f \geq 0,\ x_{i,t}^f \geq 0,\ l_{i,t}^f \geq 0, \tag{54}$$

$$A_t = \sum_{f=1}^{F} m^f \int_i a_{i,t}^f \ell^f(di),\ L_t = l_t \sum_{f=1}^{F} m^f \int_i (y_{i,t}^f)^{\frac{1/\varphi+1+\tilde{\tau}_t}{1/\varphi+1}} \ell^f(di). \tag{55}$$

Once the previous program has been solved, we can recover τ_t, w_t, $l_{i,t}^f$ and $c_{i,t}^f$ from the resulting allocation. The constraints guarantee that the governmental budget is balanced in (51) and that the planner actually selects a competitive equilibrium characterized by individual budget constraints (52), individual Euler equations (53), individual credit and positivity constraints (54), and market clearing conditions (55).

In Section 3, we derived the planner's FOCs using a primal approach, where prices are substituted in using the FOCs of households (e.g., as in Bhandari et al., 2021). Here, we use the factorization approach, based on Marcet and Marimon (2019) and developed

---

[22] The condition $x_{i,t}^f \geq 0$ and $l_{i,t}^f \geq 0$ imply $c_{i,t}^f > 0$.

in LeGrand and Ragot (2022a). Both methods provide the same FOCs, as we show in Section A.6.2. However, the factorization approach is better suited to the interpretation and the resolution of the general case. The goal of Marcet and Marimon (2019) is to provide a recursive formulation for optimization problems with forward looking constraints (which here are the Euler equations of unconstrained agents). At the beginning of their construction (see equations (5) and (6) of Marcet and Marimon, 2019), they show that one can write the Lagrangian and then manipulate the terms to maximize the discounted sum of a single term. This term embeds forward-looking constraints and has no expectation term. This is the first step before writing a recursive formulation. We do not use the recursive formulation in our paper and only derive FOCs of the sequential problem. We thus avoid the question of the existence of a Bellman equation for the planner, where the Lagrange multipliers on the Euler equations would be state variables.

We denote as $\beta^t \lambda_{i,t}^f$ the Lagrange multiplier on the period-t Euler equation (53) of agent i of type f. When the credit constraint of agent i is binding, we have $a_{i,t}^f = -\bar{a}$ and $\lambda_{i,t}^f = 0$ because the Euler equation is not a constraint. When the credit constraint does not bind, the equilibrium can feature either $\lambda_{i,t}^f > 0$ or $\lambda_{i,t}^f < 0$ depending on whether the agents save too much or too little (from the planner's perspective). Similarly, we denote by $\beta^t \mu_t$ the Lagrange multiplier on the government budget constraint (51).

To save space, we derive the planner's FOCs in Appendix A.6, and provide the main results here. Note that we follow the literature and assume that the solution is interior and that the planner's FOCs are sufficient to characterize the optimal allocation. We provide some quantitative checks below.

To simplify the interpretation of the FOCs of the Ramsey program, we introduce the marginal social valuation of liquidity for agent i of type f defined as:

$$\psi_{i,t}^f := \omega^f u'(x_{i,t}^f) - (\lambda_{i,t}^f - (1 + r_t)\lambda_{i,t-1}^f) u''(x_{i,t}^f). \tag{56}$$

This complex expression has a simple interpretation. It is the value for the planner of transferring one unit of resources to agent i of type f (if possible). First, the extra unit is valued by the marginal utility weighted with the proper weight, $\omega^f u'(x_{i,t}^f)$. Second, this extra unit of resources also affects the savings incentives, both from period $t-1$ to t (the term in $\lambda_{i,t-1}^f$) and from period t to $t+1$ (the term in $\lambda_{i,t}^f$). These last two effects are weighted by the variation in marginal utility of consumption, $u''(x_{i,t}^f)$.

From equation (56), we also define the marginal value of the public funds financed by agent (i,f):

$$\hat{\psi}^f_{i,t} := \mu_t - \psi^f_{i,t}. \tag{57}$$

This is the net value for the planner of transferring one unit of resources to its budget from an agent (i,f) .With this notation, the FOCs of the planner are easily interpreted. First, for an unconstrained agent (i,f), the planner implements a public-funds smoothing condition:

$$\hat{\psi}^f_{i,t} = \beta \mathbb{E}_t[(1+r_{t+1})\hat{\psi}^f_{i,t+1}], \tag{58}$$

where because of the assumption of MIT shocks, the expectation is taken with respect to the idiosyncratic risk. Equation (58) is a generalized version of the Euler equation (10) (and is actually the same equation when all Lagrange multipliers are zero and all weights are set to 1), in which the planner internalizes through $\hat{\psi}^f_{i,t}$ the general equilibrium externalities when setting individual savings. For credit-constrained agents, we have $\lambda^f_{i,t} = 0$, and the Euler equation is not a constraint.

Here we present FOCs related to the fiscal tools. The FOC with respect to public debt can be written as

$$\mu_t = \beta(1+\tilde{r}_{t+1})\mu_{t+1}, \tag{59}$$

without an expectation operator because of the MIT shock assumption. Equation (59) shows that the planner aims at smoothing the shadow cost of the government budget constraint through time. This yields the modified golden rule at the steady state, as previously discussed.

The other FOC with respect to the post-tax interest rate captures the effect of a change in the capital tax:

$$\underbrace{\sum_{f=1}^{F} m^f \int_i \hat{\psi}^f_{i,t} a^f_{i,t-1} \ell^f(di)}_{\text{Net distributive gain}} = \underbrace{\sum_{f=1}^{F} m^f \int_i \lambda^f_{i,t-1} u'(x^f_{i,t}) \ell^f(di)}_{\text{Cost on savings incentives}}. \tag{60}$$

A change in the capital tax generates benefits for the government through the taxation of heterogeneous households. Because the capital tax is levied on agents' asset holdings, the benefits are proportional to their beginning-of-period wealth, which is the net distributive effect (which is the term at the left-hand side (LHS)). These benefits are set equal to the costs, which operate through the savings incentives. From the planner's perspective, these costs depend on the Lagrange multiplier $\lambda^f_{i,t-1}$ on the Euler equation of each agent (term at the right-hand side (RHS)).

The FOC capturing the effect of a change in the labor supply l_t is:

$$\underbrace{\frac{1+1/\varphi}{\chi\tilde{\tau}_t} l_t^{1/\varphi+1} \sum_{f=1}^{F} m^f \int_i \hat{\psi}_{i,t}^f (y_{i,t}^f)^{\tilde{\tau}_t} \ell(di)\ell^f(di) =}_{\text{Net welfare effect}} \tag{61}$$

$$\underbrace{\mu_t \left(\sum_{f=1}^{F} m^f \int_i \left(\frac{l_t^{1/\varphi+1}}{\chi} (y_{i,t}^f)^{\tilde{\tau}_t} - (y_{i,t}^f)^{\frac{1/\varphi+1+\tilde{\tau}_t}{1/\varphi+1}} F_{L,t} l_t \right) \ell^f(di) \right)}_{\text{Reduction in goverment income}}. \tag{62}$$

As in FOC (60), the benefit of setting the labor tax level consists in public-funds transfers weighted by the tax base, which here is the labor supply, equal to $\frac{1}{\chi\tilde{\tau}_t} l_t^{1/\varphi+1} (y_{i,t}^f)^{\tilde{\tau}_t}$, for each agent (LHS). The cost is related to the modification of labor supply incentives that are affected by labor tax (RHS).

The FOC for the progressivity coefficient $\tilde{\tau}_t$ has a similar interpretation:

$$0 = \underbrace{\frac{l_t^{1+1/\varphi}}{\chi\tilde{\tau}_t} \sum_{f=1}^{F} m^f \int_i \hat{\psi}_{i,t}^f (y_{i,t}^f)^{\tilde{\tau}_t} \left(-\frac{1}{\tilde{\tau}_t} + \log y_{j,t}^f \right) \ell(di)}_{\text{Net distributive gain}} \tag{63}$$

$$\underbrace{- \mu_t \frac{l_t}{1/\varphi+1} \left(\sum_{f=1}^{F} m^f \int_i \log y_{j,t}^f \left(\frac{l_t^{1/\varphi}}{\chi} (y_{i,t}^f)^{\tilde{\tau}_t} - (y_{i,t}^f)^{\frac{1/\varphi+1+\tilde{\tau}_t}{1/\varphi+1}} F_{L,t} \right) \ell(di) \right)}_{\text{Cost on labor supply incentives}}.$$

Setting the progressivity of the labor tax is very similar to setting its level. Indeed, on the one hand, benefits are public-funds transfers weighted by the tax base. On the other hand, the costs are related to the modification of labor supply incentives. However, even though setting the average tax level or the progressivity (coefficient τ_t) has similar effects, they are two independent instruments because they affect the distribution of agents differently.

4.2 Inverse Optimal Approach at the Steady State

A quantitative simulation of the model requires taking a stand on the SWF, which is determined by the social weights $(\omega^f)_{f=1,\dots,F}$. Indeed, our goal is to study the dynamics around a quantitatively relevant steady-state fiscal system. This is typically not the case when the steady state corresponds to a utilitarian planner with a standard calibration. To overcome this difficulty, we implement an inverse optimal approach (Bourguignon and Amadeo, 2015; Chang et al., 2018; Heathcote and Tsujiyama, 2021), allowing us to estimate the weights of the SWF. The inverse optimal approach consist in identifying the objective of the planner, for which an observed competitive equilibrium (i.e., observed allocation and instrument values) is socially optimal. More precisely, we calibrate the parameters of the model to obtain a realistic steady-state allocation given fiscal parameters set to match the actual US fiscal policy. Second, we find the values of the social welfare weights $(\omega^f)_{f=1,\ldots,F}$, such that the chosen fiscal parameters are actually optimal for the planner at the steady state, and are a solution of an SRE. Once the social weights have been obtained, we can use this steady-state allocation to implement public spending shocks (with different persistences) to observe the responses of fiscal instruments. As these shocks are transitory, we can check that the value of the fiscal tools return to their initial values, which are the optimal ones in the long run. Overall, the FOCs of the planner are used twice: (i) at the steady state, to estimate the weights of the SWF from the actual fiscal system; (ii) in the dynamics to compute the optimal response of instruments given the estimated weights.

How many SWF weights can be identified from the steady-state FOCs of the planner? Fiscal policy is composed of four instruments $(\tau^K, B, \kappa, \tau)$, but these four instruments actually impose only two constraints on social weights. Indeed, fiscal policy is constrained by the budget constraint of the government, which removes one degree of freedom. Moreover, the public debt FOC (59) imposes a steady-state value on the before-tax real interest rate $1+\tilde{r} = 1/\beta$, but does not restrict the social weights. As the social weights are unique up to an increasing transformation, we further impose without loss of generality that the weights sum up to 1: $\sum_{f=1}^{F} m^f \omega^f = 1$. Given this normalization and the two FOC constraints, $F = 3$ different types of agents are needed to exactly identify the SWF weights from the FOCs of the planner. We will thus consider $F = 3$ in our quantitative exercise of Section 5.[23]

4.3 Consistency with the Analytical Model

After constructing the SRE using the inverse optimal approach, it can be checked that the conditions identified in the discussion of Proposition 2 are satisfied in the general model. First, the social weights ω^f are constructed so that the planner's FOCs hold. In this SRE we can check that the Lagrange multiplier of the government's budget is positive and that the Straub-Werning condition holds. Second, since the government's budget constraint holds with an interior fiscal policy, the Laffer condition is satisfied. Third, we also verify that the total return on the capital is not sufficiently large to finance public

---

[23] Considering $F > 3$ types is possible, at the cost of additional restrictions, such a the minimal deviation to the Utilitarian SWF. This is done in Section 5.4 below.

spending, and therefore that a first-best equilibrium cannot exist for the calibration under consideration. These three conditions correspond to those listed in Proposition 2 about the equilibrium. Furthermore, since the calibration implies a positive public debt, the condition in Proposition 5 also holds. Finally, when we simulate the dynamics of the model, we check that the Blanchard-Kahn condition holds for the truncated model, similarly to the condition in Proposition 4. These checks make us confident that we consider the perturbation of a relevant SRE.

Identifying the price externality. A result of the analytical model, and of Proposition 1, is that externalities of savings and labor choices on prices are key to pin down the optimal fiscal system. In the resolution of the general model (see Section 4.1), we rely on the Lagrangian approach to compute the planner's FOCs. We verify in Appendix A.6.2.1 that the FOCs derived with the Lagrangian or the primal approaches are identical, even though the Ramsey problems are solved differently.[24] With this approach, the price externality is captured by the Lagrange multipliers $\lambda_{i,t}$ indicating whether agent i is saving too much or too little from a social perspective. In the absence of price externalities, agents' private savings decisions would also be socially optimal and $\lambda_{i,t} = 0$ for all agents i (i.e., the Euler equation would not be a constraint for the planner but a redundant optimality condition). As in the analytical model, the absence of price externalities implies a zero optimal capital tax. Indeed, when $\lambda_{i,t} = 0$ for all agents i, equation (56) then implies that $\psi_{i,t}^f = \omega^f u'(x_{i,t}^f)$, which, for unconstrained agents, yields with equation (58):

$$\mu_t - \omega^f u'(x_{i,t}^f) = \beta \mathbb{E}_t R_{t+1} \left(\mu_{t+1} - \omega^f u'(x_{i,t+1}^f) \right).$$

This simplifies into $\mu_t = \beta R_{t+1} \mu_{t+1}$ using the Euler equation of agent i and the MIT shock assumption. Equation (59) then implies that pre- and post-tax rates are equal to each other: $1 + \tilde{r}_t = R_t$, which means a zero-capital tax: $\tau_t^K = 0$. Obviously, no stationary equilibrium would exist in that case. Indeed, savings would diverge and marginal utilities would tend to 0, implying a non-stationary equilibrium (see Chamberlain and Wilson, 2000).

Furthermore, we can prove that a positive capital tax comes with binding credit constraints at the steady state, as is the case in the simple model. Indeed, integrating the

---

[24] Additionally, in Appendix A.6.2.2, we check that the solution of the analytical approach is quantitatively similar to the limit of the solution of the general approach when the transition matrix converges to the anti-diagonal matrix of Assumption A (see Figure 2 in Appendix).

Euler equations (53) of all agents yields, at the steady state:

$$\tau^K = \frac{\sum_{f=1}^{F} m^f \int_i \nu_i^f \ell^f(di)}{(1-\beta)\sum_{f=1}^{F} m^f \int_i u'(x_i^f)\ell^f(di)}, \tag{64}$$

where $\nu_i^f \geq 0$ is the Lagrange multiplier on an individual's credit constraint. Equation (64) shows that $\tau^K > 0$ when a positive mass of agents face a binding credit constraint at the steady state. As a consequence, having credit constraints that occasionally bind is a necessary condition for a positive optimal capital tax in an SRE.

4.4 Time-Inconsistency: Time-0 and Timeless Perspectives

In this general model, optimal policies are time inconsistent. This can be seen in the planner's FOCs, where past values of Lagrange multipliers $\lambda_{i,t-1}^f$ appear (see equations (56) used in (58)). In period 0, these Lagrange multipliers are typically initialized to zero. This means that in period 0, even in the absence of any shock, the planner is not bound by any past commitments. Obviously, this differs from the steady state, where past commitments matter (and past Lagrange multiplier values differ from zero). Thus, the planner has different incentives in period 0 than in the steady state and therefore deviates from the steady-state allocation. This is called a reoptimization shock and involves the time inconsistency of the planner's program in period 0.[25] Since we do not want our results to be affected by this effect, we neutralize the time inconsistency by setting the values of the Lagrange multipliers in period -1 to their steady-state values. This means that the planner faces the same commitments as in the steady state and it removes their incentives to deviate. In this case, in the absence of a shock, the economy optimally remains at its steady-state equilibrium.[26]

4.5 Numerical Tools

We solve the model using the tools of LeGrand and Ragot (2022a). This so-called truncation method generates a large but finite state space, allowing one to easily estimate the weights

---

[25] We study this time inconsistency and the reoptimization shock in LeGrand and Ragot (2023). A reoptimization shock (i.e., setting the Lagrange multipliers to 0 in period 0) generates a transitory dynamics even in the absence of external shock. In the case of the first-order perturbation, we check that for any variable, adding the IRF of a pure reoptimization shock to the IRF of a shock on G in a timeless perspective (i.e., with no reoptimization) exactly generates the IRF of a shock on G in the time-0 perspective (i.e., with a shock on G plus a reoptimization shock).

[26] An additional benefit of this procedure is that the implied IRFs can be thought as the IRFs of a model with aggregate risk, where we take a first-order approximation of the model for the aggregate risk.

of the SWF and simulate the dynamics of the model. LeGrand and Ragot (2022b) propose a refinement of the truncation method, which we improve in this paper to allow for a refined truncation with an arbitrarily large number of different productivity levels. The formal algebra is detailed in Appendix A.7.

The truncation method can be summarized as follows. It consists of aggregating the model according to agents' recent idiosyncratic histories, and then expressing the model in terms of these groups of agents rather than individual agents. Indeed, in heterogeneous-agent models, agents differ according to their idiosyncratic histories. An agent i has a period-t history $(y_{i,0}, \ldots, y_{i,t})$. Let $h = (\tilde{y}_{-N+1}, \ldots, \tilde{y}_{-1}, \tilde{y}_0)$ be a given history of length N. In period t, an agent i is said to have *truncated history* h if the history of this agent for the last N periods is equal to h: $(y_{i,t-n+1}, \ldots, y_{i,t}) = (\tilde{y}_{-N+1}, \ldots, \tilde{y}_{-1}, \tilde{y}_0)$. The truncation method then consists of constructing a model based on these truncated histories, which serve as representative agents. The difficulty in the aggregation is that the steady state of the Bewley model features a distribution of agents within each truncated history (according to the history of agents prior to period $t - N$). It can be shown that this within-history heterogeneity can be captured by history-specific parameters (denoted by "ξs"). The truncation method assumes that this within-history heterogeneity is time-invariant and thereby allows the simulation of the dynamics.[27]

The previous truncation method considers truncated histories of equal length. This provides simplicity but at the cost of considering many histories, some of which are very unlikely to be experienced by agents. LeGrand and Ragot (2022b) proposed to consider different truncation lengths for different histories; for clarity, we call this method *refined* truncation and the former one *uniform* truncation. Histories that are more likely to be experienced (i.e., larger histories) can be "refined", i.e., that they can be replaced by a set of histories with a higher truncation length. For instance, the truncated history (y_1, y_1) can be refined into $\{(y, y_1, y_1) : y \in \mathcal{Y}\}$, where the group of agents who have been in productivity y_1 for two consecutive periods is divided into $Card(\mathcal{Y})$ truncated histories, depending on their productivity status 3 periods ago. The construction is recursive because the set $\{(y, y_1, y_1) : y \in \mathcal{Y}\}$ contains the truncated history (y_1, y_1, y_1) that can refined in a similar way. An advantage of this construction is that the number of histories is a *linear* function of the maximum truncation length, instead of an exponential function. A

---

[27] Considering wealth bins is not possible because the savings function and thus the transitions across wealth bins are endogenous to the planner's policy. This would imply a fixed point that would be very hard to solve. LeGrand and Ragot (2022a) showed that the truncated allocation converges to the true one when the truncation length increases. The question of the truncation length is then quantitative, and LeGrand and Ragot (2023) showed that a tractable truncation length provides accurate results.

difficulty of the construction is that the set of refined histories must form a well-defined partition of the set of idiosyncratic histories in each period. The construction of the refinements is presented in Appendix A.7.1. One can check the accuracy of the refined truncation, simulating economies where other solution techniques can be used. This is done in Section A.8, where we use Reiter's (2009) method as a benchmark.

5 Numerical Analysis

5.1 Calibration

Preferences. The period is a quarter. The discount factor (together with the technology parameters) is set to match an annual capital-to-output ratio of 2.7, a standard US estimate. For the log-GHH autility function (32), we set a Frisch elasticity of the labor supply of $\varphi = 0.5$, which is the value recommended by Chetty et al. (2011) for the intensive margin in heterogeneous-agent models. The scaling parameter is set to $\chi = 0.05$ to obtain a steady-state labor supply of roughly 1/3.

Ex-ante types of agents. As explained in Section 4.2, we consider three types of agents that differ ex-ante. Each type of agent is endowed with its own productivity process. The three types will enable the model to replicate at the steady state the actual fiscal policy of the US.

Productivity and idiosyncratic risk. We first use data on US educational attainment to determine the average productivity levels. We set the relative average productivity levels based on the average annual earnings of three groups of workers: those with a high-school degree or less, those with some college education and no college degree or associate degree, and those with at least a bachelor's degree. This leads us to set the relative average productivity levels of the three types to 0.8, 1 and 2, and their corresponding population shares to 1/3 for each type.[28] Type 1 is the type with the lowest average productivity, while Type 3 corresponds to the highest average productivity.

For the labor market process for each type of agents,we follow the strategy of Castaeneda et al. (2003), which is to fit realistic processes based on targeted moments. First, we focus on standard AR(1) processes for each type f: $\log y_t^f = \rho_y^f \log y_{t-1}^f + \varepsilon_t^f$, where

---

[28] Using the 2022 Current Population Survey, the three groups each represent roughly 1/3 of the US labor force, and have average annual incomes of \$24400, \$31000 and \$71000 respectively.

$\varepsilon_t^f \overset{\text{iid}}{\sim} \mathcal{N}(0, (\sigma_y^f)^2)$. Second, we perform a grid search to minimize the distance between the three processes, while imposing the following constraints: i) consistent with US data (see LeGrand and Ragot, 2018), the agents with the lowest average income face a higher income risk, ii) we target a realistic debt-to-GDP ratio given the chosen fiscal system, and iii) all social weights must be positive. We impose this last constraint to obtain a sensible SWF (in which the planner positively cares for all agents' types). The resulting parameters are gathered in Table 1.

Finally, we discretize each productivity process using the Rouwenhorst (1995) procedure considering five idiosyncratic states for each process. We thus have $5 \times 3 = 15$ productivity levels in the economy.

Technology. The production function is Cobb–Douglas: $F(K, L) = K^\alpha L^{1-\alpha} - \delta K$. The capital share is set to $\alpha = 36\%$ and the depreciation rate to $\delta = 2.5\%$, as in Krueger et al. (2018) among others.

Taxes and government budget constraint. The capital tax is taken from Trabandt and Uhlig (2011), who used the methodology of Mendoza et al. (1994) on public finance data prior to 2008. Their estimation for the US in 2007 (before the financial crisis) yields a capital tax (including both personal and corporate taxes) of $\tau^K = 36\%$. For labor, we consider the HSV functional form from equation (2). The progressivity of the labor tax is taken from Heathcote et al. (2017), who reported an estimate of $\tau = 0.18$. We choose κ to match a public-spending-to-GDP ratio of 17%, as in Heathcote and Tsujiyama (2021).

Table 2 summarizes the model parameters.

5.2 Simulation, Truncation and Estimating SWF Weights

To construct the finite state-space representation, we first use a uniform truncation length of $N = 2$ for each agent, thus generating $15^2 = 225$ histories. Second, we refine the 15 most common histories with a truncation length of 10. This means that the total number of histories is 455. This representation provides an accurate simulation of the dynamics, as shown in Appendix A.8, where we compare the dynamics of the economy simulated with the truncation and the Reiter methods, after a public spending shock for exogenous fiscal rules.

In Appendix A.7, we provide a detailed account of the computational implementation, which is of independent interest because solving such Ramsey problems is not straightforward. To summarize, the truncation method provides a finite state-space representation, which is used to compute steady-state Lagrange multipliers λ_h and social value of liquidity ψ_h for all histories $h = 1, \ldots, 455$. We then use the method of Section 4.2 to compute the social weights. Because of the two restrictions imposed by the planner's FOCs and the normalization condition, the computation of the SWF weights boils down to the inversion of a 3×3 matrix. The three social weights are found to be: $\omega_1 = 13.1\%$, $\omega_2 = 81.6\%$ and $\omega_3 = 5.3\%$. They are positive and sum to 100% by normalization. We recall that by construction, the chosen fiscal system is optimal for the planner at the steady-state.[29]

5.3 Model Dynamics

We now simulate the optimal dynamics of the four fiscal tools $(\tau_t^K, B_t, \kappa_t, \tau_t)_{t \geq 0}$ after a public spending shock occurring in period $t = 0$. After an initial shock denoted ϵ_0 in period 0, public spending reverts back to its equilibrium value at rate ρ_G. The dynamics of public spending are: $G_0 = (1 + \epsilon_0) G_{ss}$ and $G_t = (1 - \rho_G) G_{ss} + \rho_G G_{t-1}$.

Dynamics of the instruments as a function persistence. We simulate the model for two values of the persistence of public spending shocks. The higher value, $\rho_G = 0.99$, corresponds to a very persistent shock. The lower value is $\rho_G = 0.1$ and corresponds to a transitory shock. The initial size of the shock is adjusted so that the NPV of public spending is the same in the two economies. Results are plotted in Figure 1, which reports the public spending shock G, the Lagrange multiplier μ, and optimal public debt B in proportional deviations, and the labor tax level κ, the labor tax progressivity τ, and the capital tax τ^k in level deviations. The high-persistence economy is plotted with dashed lines, while the low-persistent one is plotted with solid lines. The thin red dashed line indicates the zero value (i.e., the steady-state value).

FIGURE 1 HERE

Panel 1 presents the dynamics of public spending, G, which increases by 1% of GDP when $\rho_G = 0.1$ (solid line) and by 0.02% of GDP when $\rho_G = 0.97$ (dashed line). These two different date-0 increases are calculated so that the NPV of public spending is the same

---

[29] We have checked that the social weights move intuitively as a function of the steady-state allocation. For instance, decreasing τ from 0.18 to 0.10 (recall that $\tau = 0$ is a linear fiscal system) and increasing κ to 0.76 to balance the government budget increases the social weight of the most productive agents ($\omega_1 = 5.64\%$, $\omega_2 = 0.70\%$ and $\omega_3 = 0.24\%$). We also checked that increasing the truncation length did not significantly change the weights.

in both economies. Panel 6 plots the value of the Lagrange multiplier μ (in proportional deviations), which represents the marginal value of additional public resources. Panels 2–4 report the level of the labor tax, κ, the progressivity of the labor tax, τ, and the capital tax τ^k (in level deviations). Recall that the tax schedule (2) is such that the post-tax wage is $w_t = \kappa_t(\tilde{w}_t)^{1-\tau_t}$. Therefore, an increase in κ (panel 2) corresponds to a decrease in the labor tax (as agents receive more labor income), while an increase in τ (panel 3) implies a more progressive labor tax.

First, after the public spending shock, the capital tax increases (panel 4), and the planner reduces the labor tax (panel 2) and increases its progressivity (panel 3) to reduce income inequality. Note that the change in the capital tax (panel 4) is an order of magnitude larger than the change in the labor tax. Moreover, the higher the persistence, the smaller the change in these variables. However, the variation of taxes at impact as a function of persistence is much lower than the variation in public spending. Indeed, while the tax paths are quite similar for the two persistence levels, the changes in the public debt path (panel 5) are quite different. Public debt increases when the persistence is low, making it easier to finance the sharp increase in public spending in the early periods. In contrast, public debt decreases when the persistence is high, as the cost of the additional public spending is front-loaded. These responses of the public debt explain why the variations in the tax responses, functions of the persistence of the public spending shock, are not commensurate with the variations in the date-0 shock.

To summarize, in both cases (high and low persistence), the planner implements a significant increase in capital taxes for a few quarters. Labor taxes move much less, with a small decrease in the overall level and a small increase in progressivity. Public debt shows much more persistent deviations than other variables do. Moreover, it can either fall or rise depending on the persistence of the public spending shock. This confirms the robustness of our theoretical result in Section 3.3, in a quantitatively relevant setting.

Allocation and comparison with the first-best outcome. We now compare the outcomes of the incomplete market model to those of the first-best allocation. The first-best allocation is computed in the complete market economy, in which the planner maximizes aggregate welfare subject only to the resource constraint. The first-best allocation implicitly assumes that the planner has access to productivity-contingent lump-sum taxes, as in the standard real business-cycle model. The weights of the SWF do not affect the dynamics of aggregate quantities in this case, but only the intra-period allocation.

We compare the incomplete market allocation to the first-best one both in terms of aggregate quantities and in terms of per-period welfare expressed in equivalent consumption. The latter is computed as follows. For each type of agent $f = 1, 2, 3$, there are N_{tot} histories indexed by h. We denote by $c^f_{h,t}$ and $l^f_{h,t}$ the consumption and labor supply of agents with history h of type f in period t. Their period welfare, W_t, can be computed as follows:

$$W_t = \sum_{f=1}^{3} \omega^f m^f \sum_{h=1}^{N_{tot}} S^f_h \xi^f_{0,h} u\left(c^f_{h,t} - \frac{1}{\chi} \frac{(l^f_{h,t})^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right),$$

where S^f_h is the share of the population of type f with history h and the parameter $\xi^f_{0,h}$ captures the steady-state heterogeneity within history h of type f. More precisely, $\xi^f_{0,h}$ ensures that the steady-state period utility of each history is equal to the utility derived from steady-state consumption $c^{f,ss}_h$ and labor supply $l^{f,ss}_h$: $\xi^f_{0,h} u(c^{f,ss}_h \Delta_t - \frac{1}{\chi} \frac{\left(l^{f,ss}_h\right)^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}})$ is exactly the steady-state utility of agents with history h of type f in the full model. Using these elements, we can then compute the per-period equivalent consumption, Δ_t, defined as the increase in the steady-state consumption of all agents at time t that makes each agent's period welfare identical to the period welfare W_t. Formally:

$$\sum_{f=1}^{3} \omega^f m^f \sum_{h=1}^{N^{tot}} S^f_h \xi^f_{0,h} u\left(c^{f,ss}_h \Delta_t - \frac{1}{\chi} \frac{\left(l^{f,ss}_h\right)^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right) = W_t.$$

For the first-best case, the calculation is similar except that there is only one agent (hence one type and one history).

FIGURE 2 HERE

The results are plotted in Figure 2, where we report output Y, capital K, aggregate labor supply L, aggregate consumption C, and welfare (in equivalent consumption as explained above) after a public spending shock. Panel A reports results for the low-persistence case ($\rho_G = 0.1$), and panel B reports results for the high-persistence case ($\rho_G = 0.99$). The public spending shock is shown in panel 1 of Figure 5 for the two persistence values. In both panels of Figure 2, the solid line corresponds to the incomplete market model (IM) and the dashed line to the first-best allocation (FB). The two economies experience the same public spending shock, which differs only in its persistence.

Consumption and the capital stock fall in all cases, but much more so when persistence is low (due to the larger shock at impact). Total labor supply increases at impact (due to a decrease in labor taxes). It can be observed that the volatility and the persistence of the aggregate variables in the incomplete-market economy are higher than in the first-best economy, for both low and high persistence values, although the dynamics of the variables are qualitatively similar.[30] The relative discrepancy between the two economies is greater in the case of high persistence case than in the case of low persistence. Finally, and unsurprisingly, the decline in welfare is significantly more pronounced in the incomplete-market economy than in the first-best economy. Moreover, the welfare gap between the two economies decreases faster in the low persistence case than in the high persistence case.

Optimal path of public debt and persistence of the shock. As we saw in Section 3.3, and as confirmed by the quantitative analysis reported in panel 5 of Figure 1, the response of the public debt differs markedly with the persistence of the shock. We explore this aspect further here by reporting the optimal debt dynamics for four different levels of persistence of the public spending shock. This is done in Figure 3, where in each case the initial shock G_0 is normalized to produce the same NPV of public spending. The paths of other instruments or other aggregate quantities are similar to those presented in Figure 1 and are therefore not reported here.

FIGURE 3 HERE

We observe that the response of the public debt at impact decreases with persistence. When the persistence is small ($\rho_G = 0.1$), public debt on impact first increases and then decreases monotonically. The response at impact decreases monotonically with persistence. The shape of the response also changes. For higher persistence ($\rho_G = 0.8$), the path of public debt has an inverted U shape that then becomes J-shaped at higher persistence ($\rho_G = 0.95$).

The takeaway from this graph is that the persistence of the public spending shock is a key driver of the optimal financing structure of the shock. The higher the persistence, the more the financing should rely on taxes and the less on public debt.

---

[30] It is also possible to compute the dynamics of the allocation with complete markets (representative-agent case) but with distorting taxes. It is known (from Chari et al., 1994, Chari and Kehoe, 1999, and Farhi, 2010, among others) that the optimal steady-state outcome features (i) a null capital tax, (ii) a government that holds the whole capital stock (public debt thus being negative), and (iii) a labor tax set to finance the share of public spending that is not financed by interest payment on the capital stock. After a public spending shock, public debt follows the capital stock. Because this outcome is very different from the incomplete-market economy (where steady-state public debt is positive), we do not report the simulation of this economy.

5.4 Robustness in Other Environments and Other Shocks

We check the robustness of the result concerning the optimal response of public debt in two other environments.

5.4.1 Alternative Fiscal System

Our benchmark tax system features a non-linear HSV labor tax. However, this is not the only way in the literature to reproduce the progressivity of the US fiscal system. Another possibility is to consider an affine tax system, in which the linear labor tax is complemented by a lump-sum transfer T_t. This is the case, for instance in Dyrda and Pedroni (2022). We present the model specification and the solution of the Ramsey program in Appendix A.9. We verify that our main result remains robust to this new fiscal system. We find that public debt increases when persistence is low but decreases when persistence is high. In addition, similarly to the quantitative results in Figure 1, both the tax progressivity and the capital tax increase at impact. Overall, the optimal response of the fiscal system remains robust to the specification of the tax system.

5.4.2 Alternative SWF

The benchmark SWF assigns social weights that depend on the ex-ante type of agents. The ex-ante types were fixed once and for all, and implied different productivity processes among agents. We then used these weights to implement an inverse optimal taxation approach: the weights were calibrated for the actual US tax system to be optimal at the steady state. Here we consider an alternative SWF, where the weights are productivity dependent. We assume that agents can draw their current level of productivity within a given finite set and the planner then assigns a social welfare weight, $\omega(y)$, to each level of productivity, y. The instantaneous utility of agents with productivity level y in the current period is weighted by $\omega(y)$. Even if the weights are set once and for all, a given agent may experience different weights depending on their current productivity level. Formally, the planner's SWF is:

$$\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_{i,t})(u(c_{i,t}) - v(l_{i,t}))\ell(di), \tag{65}$$

where we remove ex-ante heterogeneity. In fact, this formulation of social welfare weights does not require ex-ante heterogeneity, since it only involves within-period heterogeneity. The advantage of the representation (65), is that the planner may have a preference for within-period redistribution, since the weights depend on productivity, which may be an attractive feature. This SWF is used in LeGrand et al. (2024), Dávila and Schaab (2022) and McKay and Wolf (2023). This representation also allows considering a separable utility function, which is CRRA for consumption. However, these weights are not strictly speaking social weights, since they weight instantaneous utility but not intertemporal welfare.

The model specification and results are provided in Appendix A.10. Again, we check that the results are qualitatively similar to those of the baseline model.

5.4.3 Other Shocks

The dynamics of the model with other shocks can be simulated using the same approach as in Section 5.2. In fact, once the Ramsey steady state has been characterized, the method for simulating the dynamics of the model is very versatile and can be easily adapted to different shocks. We illustrate this here by considering a TFP shock and a discount factor shock that complements the aforementioned public spending shock.

Regarding the TFP shock, we assume that the production function is $Y_t = F(K_{t-1}, L_t) = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} - \delta K_{t-1}$, where Z_t is the TFP, equal to 1 in the steady-state. Public spending is constant. After an initial (small) shock ϵ_0, TFP returns to its equilibrium value at a rate $\rho_Z \in [0,1)$. The dynamics of TFP is therefore: $Z_0 = 1 - \epsilon_0$ and $Z_t = 1 - \rho_Z + \rho_Z Z_{t-1}$, for $t \geq 1$. We then simulate the dynamics of the model for different values of the persistence ρ_Z. As in the case of the public spending shock, we adjust the size of the initial shock ϵ_0, so that the cumulative fall in TFP is constant over the different values of ρ_Z. For the sake of brevity, the optimal dynamics of public debt for the different values of ρ_Z can be found in Appendix A.11. The results are very similar to those in Figure 3. When the TFP shock is not persistent, the public debt increases on impact, while it decreases when it is very persistent. The intuition behind this result is as follows. An increase in public spending is a reduction in the resources available for consumption (even if it increases the level of welfare), which is known to be very similar to a fall in TFP. In both cases, the planner must raise additional resources, either because spending increases or because the tax bases shrink.

Regarding the discount factor shock, the discount factor shared by agents and the planner is time-varying, while public spending and TFP remain constant. Formally, the path of the discount factor, denoted $(\beta_t)_{t\geq 0}$, is defined as $\beta_0 = \beta + \epsilon_0$ and $\beta_t = (1-\rho_\beta)\beta + \rho_\beta \beta_{t-1}$, for $t \geq 1$. The initial shock is ϵ_0 and the persistence $\rho_\beta \in [0,1)$, while β is the steady-state value of the discount factor set using the baseline calibration. Again, we consider the public debt response for different values of the persistence ρ_β and the initial shock is normalized so that the average variation in the discount factor is constant across the different persistence values. The results can be found in Appendix A.11. We find that the dynamics of the public debt is different compared to the cases of the TFP and public spending shocks. Since agents are temporarily more patient, the capital stock increases and this increase is more persistent than the discount factor shock. This allows the planner to reduce the public debt. When the persistence is very high, public debt may even increase slightly on impact, since the high persistence implies a large and sustained increase in savings.

6 Conclusion

We have studied the optimal fiscal policy after a public spending shock in a heterogeneous-agent model. Our first contribution is to clarify, in a simple environment, the conditions for the existence of steady-state equilibria that feature positive optimal capital taxation and public debt. The key friction for the existence of an equilibrium is an occasionally binding credit constraint, which provides a rationale for maintaining both a positive capital tax and positive public debt. This friction is necessary but not sufficient for the existence of the equilibrium. Obtaining a positive optimal capital tax depends on the shape of the utility function; it occurs generally for DRRA or GHH utility functions, for instance. A second result is to show that the optimal dynamics of public debt and taxes depend crucially on the persistence of the public spending shock: public debt is procyclical for low persistence but countercyclical for high persistence. We show that these results still hold in a general model where we solve for optimal fiscal policy after an MIT shock. In this model, the actual US tax system is optimally implemented at the steady state, thanks to an inverse optimal taxation approach. We find that public debt can either increase or decrease on impact depending of the persistence of public spending or TFP shocks.

Data Availability

Code replicating the tables and figures in this article can be found in LeGrand and Ragot (2024) in the Harvard Dataverse, `https://doi.org/10.7910/DVN/ZMIFAZ`.

References

Açikgöz, O., M. Hagedorn, H. Holter, and Y. Wang (2022): "The Optimum Quantity of Capital and Debt," Working Paper, University of Oslo.

Acharya, S., E. Challe, and K. Dogra (2023): "Optimal Monetary Policy According to HANK," *American Economic Review*, 113, 1741–1782.

Acharya, S. and K. Dogra (2021): "Understanding HANK: Insights from a PRANK," *Econometrica*, 88, 1113–1158.

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659–684.

——— (1995): "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting," *Journal of Political Economy*, 103, 1158–1175.

Aiyagari, S. R. and E. R. McGrattan (1998): "The Optimum Quantity of Debt," *Journal of Monetary Economics*, 42, 447–469.

Albanesi, S. and R. Armenter (2012): "Intertemporal Distortions in the Second Best," *The Review of Economic Studies*, 79, 1271–1307.

Auclert, A., M. Cai, M. Rognlie, and L. Straub (2022): "Optimal Long-Run Fiscal Policy with Heterogeneous Agents," SED Presentation.

Barro, R. J. (1979): "On the Determination of the Public Debt," *Journal of Political Economy*, 87, 940–971.

Bassetto, M. (2014): "Optimal Fiscal Policy with Heterogeneous Agents," *Quantitative Economics*, 5, 675–704.

Bassetto, M. and W. Cui (2024): "A Ramsey Theory of Financial Distortions," *Journal of Political Economy*, 132, 2612–2654.

Bayer, C., R. Luetticke, L. Pham-Dao, and V. Tjaden (2019): "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk," *Econometrica*, 87, 255–290.

Bewley, T. F. (1983): "A Difficulty with the Optimum Quantity of Money," *Econometrica*, 51, 1485–1504.

Bhandari, A., D. Evans, M. Golosov, and T. J. Sargent (2017): "Public Debt in Economies with Heterogeneous Agents," *Journal of Monetary Economics*, 91, 39–51.

——— (2021): "Inequality, Business Cycles, and Monetary-Fiscal Policy," *Econometrica*, 89, 2559–2599.

Bourguignon, F. and S. Amadeo (2015): "Tax-Benefit Revealed Social Preferences," *Journal of Economic Inequality*, 1, 75–108.

Castaeneda, A., J. Diaz-Gimenez, and J.-V. Rios-Rull (2003): "Accounting for Earnings and Wealth Inequality," *Journal of Political Economy*, 111, 818–857.

Challe, E. and X. Ragot (2016): "Precautionary Saving over the Business Cycle," *Economic Journal*, 126, 135–164.

Chamberlain, G. and C. Wilson (2000): "Optimal Intertemporal Consumption under Uncertainty," *Review of Economic Dynamics*, 3, 365–395.

Chamley, C. (1986): "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 54, 607–622.

Chang, B.-H., Y. Chang, and S.-B. Kim (2018): "Pareto Weights in Practice: A Quantitative Analysis Across 32 OECD Countries," *Review of Economic Dynamics*, 28, 181–204.

Chari, V., L. Christiano, and P. Kehoe (1994): "Optimal Fiscal Policy in a Business Cycle Model," *Journal of Political Economy*, 102, 617–652.

Chari, V. V. and P. J. Kehoe (1999): "Optimal Fiscal and Monetary Policy," in *Handbook of Macroeconomics*, ed. by J. B. Taylor and M. Woodford, Elsevier, vol. 1, chap. 21, 1671–1745.

Chari, V. V., J.-P. Nicolini, and P. Teles (2020): "Optimal Capital Taxation Revisited," *Journal of Monetary Economics*, 116, 147–165.

Chen, Y., Y. Chien, and C. Yang (2020): "Implementing the Modified Golden Rule? Optimal Ramsey Capital Taxation with Incomplete Markets Revisited," St Louis Fed Working Paper 2017-003I, Federal Reserve Bank of St Louis.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011): "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence On the Intensive and Extensive Margins," *American Economic Review*, 101, 471–475.

Chien, Y. and Y. Wen (2024): "The Ramsey Steady-State Conundrum in Heterogeneous-Agent Economies," *Journal of Economic Theory*, forthcoming.

Collard, F., M. Angeletos, and H. Dellas (2023): "Public Debt As Private Liquidity: Optimal Policy," *Journal of Political Economy*, 131, 3233–3264.

Conesa, J. C., S. Kitao, and D. Krueger (2009): "Taxing Capital? Not a Bad Idea after All!" *American Economic Review*, 99, 25–48.

Dávila, E. and A. Schaab (2022): "Welfare Assessments with Heterogeneous Individuals," Working Paper 30571, National Bureau of Economic Research.

Dávila, J., J. H. Hong, P. Krusell, and J.-V. Ríos-Rull (2012): "Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks," *Econometrica*, 80, 2431–2467.

Diamond, P. (1998): "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates," *American Economic Review*, 88, 83–95.

Dyrda, S. and M. Pedroni (2022): "Optimal Fiscal Policy in a Model with Uninsurable Idiosyncratic Shocks," *Review of Economic Studies*, 90, 744–780.

Farhi, E. (2010): "Capital Taxation and Ownership When Markets Are Incomplete," *Journal of Political Economy*, 118, 908–948.

Feldstein, M. (1985): "Debt and Taxes in the Theory of Public Finance," *Journal of Public Economics*, 28, 233–245.

Ferriere, A. and G. Navarro (2023): "The Heterogeneous Effects of Government Spending: It's All About Taxes," *Review of Economic Studies*, forthcoming.

Fishburn, P. (1977): "Mean-Risk Analysis with Risk Associated with Below-Target Returns," *American Economic Review*, 67, 116–126.

Floden, M. (2001): "The Effectiveness of Government Debt and Transfers as Insurance," *Journal of Monetary Economics*, 48, 81–108.

Green, E. (1994): "Individual-Level Randomness in a Nonatomic Population," Working Paper, University of Minnesota.

Heathcote, J. (2005): "Fiscal Policy with Heterogeneous Agents and Incomplete Markets," *Review of Economic Studies*, 72, 161–188.

Heathcote, J., K. Storesletten, and G. L. Violante (2017): "Optimal Tax

Progressivity: An Analytical Framework," *Quarterly Journal of Economics*, 132, 1693–1754.

Heathcote, J. and H. Tsujiyama (2021): "Optimal Income Taxation: Mirrlees Meets Ramsey," *Journal of Political Economy*, 129, 3141–3184.

Huggett, M. (1993): "The Risk Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, 17, 953–969.

Imrohoroğlu, A. (1989): "Cost of Business Cycles with Indivisibilities and Liquidity Constraints," *Journal of Political Economy*, 97, 1364–1383.

Judd, K. L. (1985): "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 28, 59–83.

King, R., C. Plosser, and S. Rebelo (1988): "Production, Growth and Business Cycles I. The Basic Neoclassical Model," *Journal of Monetary Economics*, 21, 195–232.

Krueger, D., K. Mitman, and F. Perri (2018): "On the Distribution of the Welfare Losses of Large Recessions," in *Advances in Economics and Econometrics: Volume 2, Eleventh World Congress of the Econometric Society*, ed. by B. Honoré, A. Pakes, M. Piazzesi, and L. Samuleson, Cambridge University Press, 143–184.

Krusell, P. and A. A. J. Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867–896.

Lansing, K. J. (1999): "Optimal Redistributive Capital Taxation in a Neoclassical Growth Model," *Journal of Public Economics*, 73, 423–453.

LeGrand, F., A. Martin-Baillon, and X. Ragot (2024): "Should Monetary Policy Care About Redistribution? Optimal Fiscal and Monetary Policy with Heterogeneous Agents," Working paper, SciencesPo.

LeGrand, F. and X. Ragot (2018): "A Class of Tractable Incomplete-Market Models for Studying Asset Returns and Risk Exposure," *European Economic Review*, 103, 39–59.

——— (2022a): "Managing Inequality Over the Business Cycle: Optimal Policies with Heterogeneous Agents and Aggregate Shocks," *International Economic Review*, 63, 511–540.

——— (2022b): "Refining the Truncation Method to Solve Heterogeneous-Agent Models," *Annals of Economics and Statistics*, 146, 65–91.

——— (2023): "Optimal Policies with Heterogeneous Agents: Truncation and Transitions," *Journal of Economic Dynamics and Control*, 156, 104737.

——— (2024): "Replication code for "Optimal Fiscal Policy with Heterogeneous Agents and Capital: Should We Increase or Decrease Public Debt and Capital Taxes?"," `https://doi.org/10.7910/DVN/ZMIFAZ`.

Marcet, A. and R. Marimon (2019): "Recursive Contracts," *Econometrica*, 87, 1589–1631.

McKay, A. and C. Wolf (2023): "Optimal Policy Rules in HANK," Working Paper, FRB Minneapolis.

Mendoza, E., A. Razin, and L. Tesar (1994): "Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption," *Journal of Monetary Economics*, 34, 297–323.

Nuño, G. and C. Thomas (2022): "Optimal Redistributive Inflation," *Annals of Economics and Statistics*, 146, 3–64.

Reinhorn, L. (2019): "On Optimal Redistributive Capital Taxation," *Journal of Public Economic*, 21, 460–487.

Reiter, M. (2009): "Solving Heterogeneous-Agent Models by Projection and Perturbation," *Journal of Economic Dynamics and Control*, 33, 649–665.

Rohrs, S. and C. Winter (2017): "Reducing Government Debt in the Presence of Inequality," *Journal of Economic Dynamics and Control*, 82, 1–20.

Rouwenhorst, G. K. (1995): "Asset Pricing Implications of Equilibrium Business Cycle Models," in *Structural Models of Wage and Employment Dynamics*, ed. by T. Cooley, Princeton: Princeton University Press, 201–213.

Slater, M. (1950): "Lagrange Multipliers Revisited," Working Paper 80, Cowles Foundation, reprinted in Giorgi G., Kjeldsen, T., eds. (2014). Traces and Emergence of Nonlinear Programming. Basel.

Straub, L. and I. Werning (2020): "Positive Long Run Capital Taxation : Chamley-Judd Revisited," *American Economic Review*, 110, 86–119.

Trabandt, M. and H. Uhlig (2011): "The Laffer Curve Revisited," *Journal of Monetary Economics*, 58, 305–327.

Woodford, M. (1990): "Public Debt as Private Liquidity," *American Economic Review*, 80, 382–388.

Tables

	Type 1	Type 2	Type 3
persistence ρ_y	0.986	0.98	0.98
Variance σ_y	0.16	0.132	0.132
Average productivity	0.8	1.0	2.0

Table 1: Model calibration: targets and model counterparts.

Parameter	Description	Value
	Preference and technology	
β	Discount factor	0.992
α	Capital share	0.36
δ	Depreciation rate	2.5%
$\bar{a}$	Credit limit	0
χ	Scaling param. labor supply	0.05
φ	Frisch elasticity labor supply	0.5
	Tax system	
τ^K	Capital tax	36%
κ	Scaling of labor tax	0.75
τ	Progressivity of tax	18%
B/Y	Public debt	61%
G/Y	Public consumption	17%

Table 2: Parameter values in the baseline calibration. See the text for descriptions and targets.

Figures

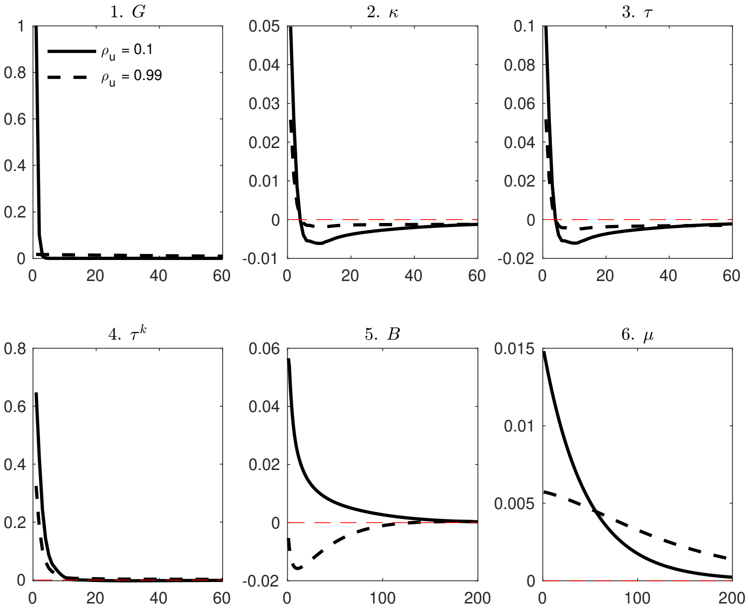
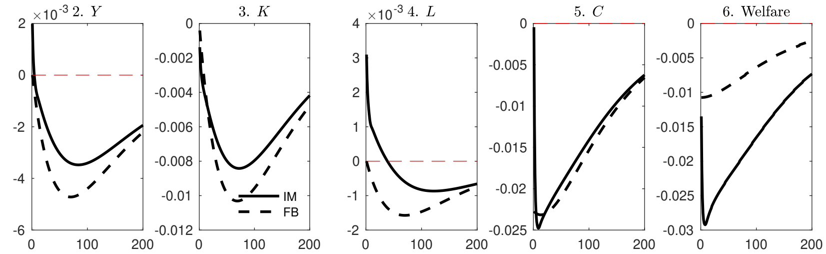


Figure 1: Dynamics of selected variables for two shocks with different persistences and the same NPV. G—public spending; μ—value of public resources; κ—level of labor tax; τ—progressivity of labor tax; τ^k—capital tax; B—public debt. The solid lines correspond to persistence $\rho_G = 0.1$, and the dashed lines correspond to persistence $\rho_G = 0.99$. G is in percent of GDP, B is in proportional deviations, and other variables are in level deviations.

A. High persistence $\rho_G = 0.99$



B. Low persistence $\rho_G = 0.1$

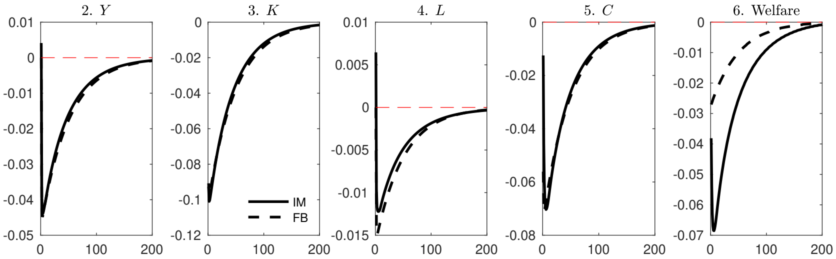


Figure 2: Output Y, capital K, labor L, consumption C, and period aggregate welfare (in equivalent consumption) for low and high persistence values, in proportional deviations. The solid line is the incomplete market model (IM) and the dashed line the first-best allocation (FB). The shock is a public spending shock (panel 1 of Figure 5) and only differs in persistence.

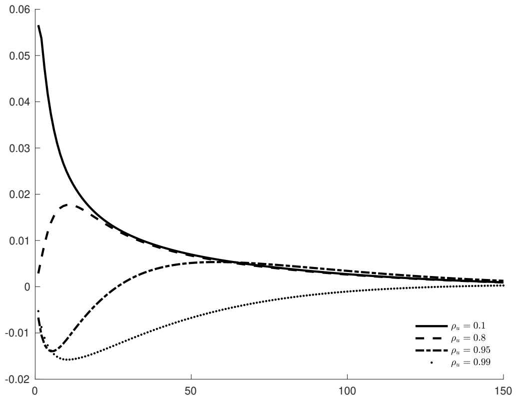


Figure 3: Optimal public debt dynamics for different persistence values of the public shock (same NPV of public spending), in proportional deviation from steady-state value of public debt.